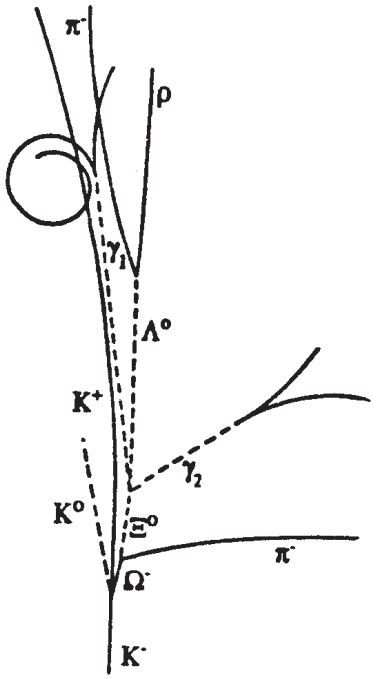
VOLUME 45, NUMBER 3, SEPTEMBER 2022

HADRONIC JOURNAL Volume 45, Number 3, September 2022

HADRONIC JOURNAL

Founded in 1978 by Prof. R. M. Santilli at Harvard University. Some of the past Editors include Professors S. L. Adler, A. 0. Barut, L. C. Biedenharn, N. N. Bogoliubov, M. Froissart, J.Lohmus, S. Okubo, Nobel Laureate Ilya Prigogine, M. L. Tomber, J. P.Vigier, J. Wess, Nobel Laureate Chen Ning Yang.



EDITORIAL BOARD

A.O.ANIMALU
A. K. ARINGAZIN
A. A. BHALEKAR
S. J. DHOBLE
J. DUNNING-DAVIES T.
L. GILL
L. P. HORWITZ
S. L. KALLA
S. I. KRUGLOV
M. NISHIOKA
R. F. O'CONNELL
Z. OZIEWICZ*
E. RECAMI
M. SALEEM
B. G. SIDHARTH
S. SILVESTROV
H. M. SRIVASTAVA
E. TRELL
R.I. TSONCHEV
QI-REN ZHANG
C.A.WOLF
YI-ZHONG ZHUO

** In memoriam*

FOUNDER and Editor In Chief
R. M. SANTILLI

HADRONIC PRESS, INC.

HADRONIC JOURNAL

Established in 1978 by Prof. R. M. Santilli at Harvard University Hadronic Journal and Algebras, Groups and Geometries have been regularly published since 1978 without publication charges and are among the few remaining independent refereed journals.

This Journal publishes advances research papers and Ph. D. theses in any field of mathematics without publication charges.

For subscription, format and any other information please visit the website
http://www.hadronicpress.com

HADRONIC PRESS INC.
35246 U. S. 19 North Suite 215
Palm Harbor, FL 34684, U.S.A.
http://www.hadronicpress.com
Email: info@hadronicpress.com
Phone: +1-727-946-0427

HADRONIC JOURNAL

Founder and Editor in Chief
RUGGERO MARIA SANTILLI
The Institute for Basic Research
35246 U. S. 19 North Suite 215, Palm Harbor, FL 34684, U.S.A.
Email: resarch@i-b-r.org; TEL: +1-727-688-3992

A. O. ANIMALU, University of Nigeria, Department of Physics, Nsukka, Nigeria animalu@nmc.edu.ng
A.K. ARINGAZIN Department of Theoretical Physics, Institute for Basic Research Eurasian National University Astana 010008, Kazakhstan aringazin@mail.kz
A.A. BHALEKAR Department of Chemistry, R.T.M. Nagpur University, Nagpur, 440033 India anabha@hotmail.com
S.J. DHOBLE Department of Physics R.T.M. Nagpur University Nagpur, 440033 India sjdhoble@rediffmail.com
J. DUNNING-DAVIES, Department of Physics (Retired) University of Hull Hull, HU6 7RX England j.dunning-davies@hull.ac.uk
T. L. GILL Howard University Research Center ComSERC Washington, DC 20059, USA tgill@howard.edu
L. P. HORWITZ, Department of Physics Tel Aviv Univ., Ramat Aviv, Israel horwitz@taunivm.tau.ac.il
S. L. KALLA, Department of Mathematics Vyas Institute of Higher Education Jodhpur, 342008, India shyamkalla@gmail.com
S.I. KRUGLOV University of Toronto at Scarborough, Physical and Environmental Sciences Dept., 1265 Military Trail, Toronto, Ontario, Canada M1C 1A4 skrouglo@utm.utoronto.ca
M. NISHIOKA, Yamaguchi University Department of Physics, Yamaguchi 753, Japan
R. F. O'CONNELL, Louisiana State University Department of Physics, Baton Rouge, LA 70803
Z.OZIEWICZ,* Universidad National Autonoma de Mexico, Faculdad de Estudios Superiores C.P. 54714, Cuautitlan Izcalli Aparto Postal # 25, Mexico **** In memoriam***
E. RECAMI, Universita' de Bergamo, Facolta' di Ingeneria, Viale Marconi 5, 1-24044 Dalmine (BG) Italy recami@mi.ifn.it
M. SALEEM, University of the Punjab Center for High Energy Phys., Lahore, Pakistan dms@lhr.paknet.com.pk
B.G. SIDHARTH International Institute for Applicable Mathematics &b Information Sciences, B.M. Birla Science Center, Adarshnagar, Hyderabad 500063 (India) iimisbgs@yahoo.com.in
S. SILVESTROV School of Education, Culture and Communication (UKK) Malardalen University Box 883, 71610 Västerås, Sweden sergei.silvestrov@mdh.se
H. M. SRIVASTAVA, Department of Mathematics and Statistics, University of Victoria, Victoria, B. C. V8W 3P4, Canada, hmsri@uvvm.uvic.ca
E. TRELL Faculty of Health Sciences, University of Linkoping, Se-581 83, Linkoping, Sweden erik.trell@gmail.com
R.I. TSONCHEV Facultad de Fisica, Universidad Autonoma de Zacatecas, P. O. C-580, Zacatecas 98068, zac, Mexico rumen@ahobon.reduaz.ms
QI-REN ZHANG, Peking University, Department of Technical Phys., Beijing 100871, China zhangqr@sun.ihep.ac.cn
C.A. WOLF Department of Physics, Massachusetts College of Liberal Arts, North Adams, Ma 01247 cwolf@mcla.mass.edu
YI-ZHONG ZHUO, Institute of Atomic Energy P.O. Box 275 (18) Beijing 102413, China zhuoyz@mipsa.ciae.ac.cn

ISSN: 0162-5519

Established in 1978 by Prof. R. M. Santilli at Harvard University Hadronic Journal and Algebras, Groups and Geometries have been regularly published since 1978 without publication charges and are among the few remaining independent refereed journals.

This Journal publishes advances research papers and Ph. D. theses in any field of mathematics without publication charges.

For subscription, format and any other information please visit the website http://www.hadronicpress.com

HADRONIC PRESS INC.
35246 U. S. 19 North Suite 215
Palm Harbor, FL 34684, U.S.A.
http://www.hadronicpress.com
Email: info@hadronicpress.com
Phone: +1-727-946-0427

HADRONIC JOURNAL Volume 45, Number 3, September 2022

HADRONIC PRESS, INC.

HADRONIC JOURNAL

VOLUME 45, NUMBER 3, SEPTEMBER 2022

PLANCK-SCALE QUANTUM FIELD THEORY-II-EVALUATION OF QED ANALOGUE AMPLITUDES, 235
J. J. Bevelacqua
Bevelacqua Resources
343 Adair Drive
Richland, WA 99352 USA

MASS FORMULA OF HADRONS FOR THREE GENERATION QUARK STANDARD MODEL AND DIFFERENCES ON ELECTROMAGNETIC MASS OF BARYON-MESONS, 253
Yi-Fang Chang
Department of Physics
Yunnan University
Kunming, 650091, China

NEGENTROPIC THERMODYNAMICS I: PROOF OF THE NEGENTROPIC NATURE OF THE TRANSFORMATION OF HEAT INTO WORK, 263
José C. Íñiguez
Retired Instructor of Science and Mathematics
1227 21st Street
Douglas, AZ 85607

NEGENTROPIC THERMODYNAMICS II: AN AUTOPOIETIC, HEAT-ENGINE-CENTERED MODEL FOR RAYLEIGH-BÉNARD CONVECTION, 297
José C. Íñiguez
Retired Instructor of Science and Mathematics
1227 21st Street
Douglas, AZ 85607

HADRONIC CONDUCTIVITY: A NEW FORM OF SUPERCONDUCTIVITY, 333
Alexander O.E. Animalu
Department of Physics and Astronomy
University of Nigeria, Nsukka
Enugu State, Nigeria

HADRONIC JOURNAL 45, 235 - 252 (2022)
DOI 10.29083/HJ.45.03.2022/SC235

PLANCK-SCALE QUANTUM FIELD THEORY-II-EVALUATION OF QED ANALOGUE AMPLITUDES

J. J. Bevelacqua
Bevelacqua Resources
343 Adair Drive
Richland, WA 99352 USA
bevelresou@aol.com

Received April 18, 2022
Revised May 5, 2022

Abstract

An initial assessment of a Planck-Scale quantum field theory identified three basic Feynman diagrams that could generate negative energies and lead to a number of effects including wormholes, black holes, negative gravity, and supraliminal velocities. This paper reviews the importance of field quantization, renormalization, gauge theories, and gauge bosons and spontaneous symmetry breaking in affecting these phenomena. In addition, initial evaluations of the amplitudes of vacuum bubble, closed loop, and coupled single vertex diagrams at the Planck-scale are presented.

Keywords: Quantum Field Theory, Virtual Particles, Negative Energy, Planck-scale effect
PACS Index: 11.10.-Z Field Theory, 03.70.+K Quantized Fields, 25.75.Dw Particle Production, 98.80.Cq Particle Theory Models

Copyright © 2022 by Hadronic Press, Inc., Palm Harbor, FL 34684, USA

1.0 Introduction

The quantum field theory (QFT) approach outlined in Ref. 1 focused on the creation of negative energy quanta at the Planck-scale and the physical implications of the associated particles and fields. Ref. 1 focused on three basic processes for the creation of these particles. These processes include: (1) vacuum bubble, (2) closed loop, and (3) two coupled single vertex (S_1) diagrams.

The initial approach did not explicitly address the nature of the QFT interactions. Associated with the Planck-scale fields is an assumed local invariance that can only be obtained by introducing a gauge field (i.e., an interaction). The properties of the interaction are largely specified by the gauge principle. The nature of these interactions at the Planck-scale is currently unknown.

Given the uncertainty, this paper incrementally advances the theory proposed in Ref. 1. This approach is necessarily limited because considerable uncertainties exist in the specification of a Planck-scale QFT. These uncertainties include, but are not limited to: (1) the nature, characteristics, and symmetries associated with the interaction, (2) characteristics of the gauge group, (3) nature of the mediating interaction bosons, and (4) characteristics of the interacting Planck-scale particles.

Using the background of Ref. 1 as a guide, an initial assessment of the three Planck-scale diagrams that could produce negative energies is provided. These assessments are necessarily limited by the uncertainties noted previously, and are not as complete or well-established as quantum electrodynamics ([2]-[5]), electroweak theory ([6]-[8]), and quantum chromodynamics ([9]-[11]).

As a prelude to the evaluation of the Figs. 1 – 3 diagrams, applicable QFT concepts that are relevant at the Planck-scale are addressed. These include field quantization, renormalization, gauge theories, and gauge bosons and spontaneous symmetry breaking.

2.0 Field Quantization

Quantization of the gauge field produces gauge bosons as mediators of the interaction. In the case of quantum electrodynamics (QED)([2]-[5]), photons are the gauge bosons. The gauge fields also satisfy a set of equations of motion. For example, the transformed field $\Psi'(x)$ and gauge field A_μ are invariant under the transformations

$$\Psi'(x) = e^{ie\phi(x)}\,\Psi(x) \tag{1}$$

$$A'_\mu(x) = A_\mu(x) + \partial_\mu \varphi(x) \tag{2}$$

This invariance only occurs if the exchanged particles have zero rest mass. This is achieved for photons, and A_μ is interpreted to be the photon field in QED. The properties of the associated interaction are largely specified by the gauge principle. This is a general property of an unbroken gauge symmetry. In general, an unbroken symmetry predicts the existence of massless exchange bosons.

In the case of the weak interaction([6]-[8]), the gauge bosons (W^+, W^-, and Z^0) are not massless. The Higgs mechanism([12],[13]) provides a possible solution to the massless contradiction associated with an unbroken symmetry. In the Higgs mechanism, the exchanged bosons are assigned a mass through spontaneous symmetry breaking without perturbing the local gauge symmetry of the photon field. More specifically, this approach does not affect renormalizability.

Renormalization is a major advantage of gauge theories. For example, gauge theories provide a successful description of the electromagnetic and weak interactions and their unification as an electroweak interaction.

3.0 Gauge Theories

Gauge theories can be classified by the properties of their associated transformations. The structure of the desired Planck-scale theory can be described by the specification of a gauge group G(n) that reflects the symmetry properties of an n dimensional group. Ref. 1 described a Planck-scale theory, but its group (e.g., Abelian or non-Abelian), symmetry characteristics, and renormalization characteristics have yet to be defined and are not known.

Conventional theories are better defined than the postulated Planck-scale theory. For example, QED is described by the group of phase factors $\exp(ie\varphi(x))$. QED is an Abelian U(1) symmetry defined by the group of all 1 x 1 Unitary matrices. In a similar manner, electroweak and strong interactions are described by the $SU(2)_L$ and $SU(3)_c$ groups, respectively. The subscript L indicated the interaction defined by the group SU(2) is left-handed.

The Planck-scale theory could be described by either these or other more complex groups (e.g., SU(5), SO(10), and E(8)). Group dimensionality (n) is important because it determines the number of gauge bosons that mediate the interaction produced by the gauge field. In particular, the number of gauge bosons in the special unitary group SU(n) and special orthogonal group SO(n) is $n^2 - 1$. Special groups are defined by n x n group matrices having a determinant of +1.

For the conventional electroweak and color interactions, these $n^2 - 1$ gauge bosons are well defined. The electromagnetic interaction is mediated by the photon. Weak interactions are mediated by the W^+, W^-, and Z^0 bosons. Eight gluons mediate the strong interaction. Although not within the scope of the Standard Model, the gravitational interaction is mediated by the graviton. However, the proposed Planck-scale interaction bosons are unknown and may include the aforementioned mediators or currently unknown quanta with yet to be defined characteristics.

4.0 Gauge Bosons and Spontaneous Symmetry Breaking

From a theoretical perspective, the gauge bosons associated with a Planck-scale gauge theory could be massless within an unbroken symmetry. Spontaneous symmetry breaking (SSB) provides a mechanism for assigning the gauge bosons mass without perturbing gauge invariance. This follows if the current understanding of gauge theory is applicable at the Planck-scale.

SSB can occur when the fundamental equations of a system acquire a symmetry, which the ground state does not have. For example, the Lagrangian density could be invariant under a particular gauge transformation. However, the vacuum (i.e., the state of lowest energy) would not possess this symmetry.

In general, both local and global symmetries could be spontaneously broken. A spontaneous broken global symmetry is inseparably associated with a massless scalar field (i.e., the Nambu-Goldstone (NB) bosons([14],[15])). Although the NB bosons could exist at the Planck-scale, the initial focus is local gauge symmetries. This will eliminate further consideration of NB bosons in this paper.

For example, local gauge symmetries describe the electroweak interaction. Only theories that are invariant under local gauge transformation can be Lorentz invariant.

In the conventional QFT description of interactions, the SSB is produced by the Higgs mechanism that requires the introduction of additive scalar fields([12],[13],[16],[17]). Higgs fields have a non-zero vacuum expectation value (VEV). A VEV is the state of minimum energy obtained from a non-zero expectation value of the Higgs field. The Higgs fields are also important because they affect the structure of the vacuum.

Following the interaction of the massless fermion and boson fields with the Higgs field, the equations of motion are altered in a manner that the fermions and bosons behave as particles with mass. The coupling of the particle and Higgs fields is proportional to the square of the particle mass.

Since the Higgs field has a finite VEV, fermions and bosons acquire a finite potential energy that is expressed as an effective mass. Within the context of conventional gauge theories, fermions and bosons acquire mass through the interaction of the Higgs field. It is assumed that the Higgs field or a more complex version of this field also produces massive particles at the Planck-scale.

The case of the electroweak interaction is particularly illustrative. Within the electroweak interaction, both symmetries ($SU(2)_L$ and $U(1)$) must be spontaneously broken. Within the scope of known interactions, only the electromagnetic and color interactions occur as consequences of an unbroken symmetry in that massless bosons (photons and gluons) are the exchanged particles mediating their interactions. The uniqueness of the electroweak interaction is that $SU(2)_L$ and $U(1)$ are spontaneously broken in a manner that a subgroup of the product of the two groups (i.e., $U(1)$) results in an unbroken symmetry. Since the $SU(2)_L$ portion of the electroweak interaction undergoes a spontaneous symmetry breaking, its W^+, W^-, and Z^0 mediators acquire mass.

A summary of the current and postulated interactions and their mediating bosons is provided in Table 1. Only the weak interaction has been experimentally verified to have massive exchange bosons (i.e., W^+, W^-, and Z^0). The X and Y bosons are also theorized to possess mass, and emerge in postulated gauge theories (e.g., SU(5)). However, the validity of SU(5) and its predictions have not been verified experimentally.

Table 1
Properties of Exchange Bosons for Various Interactions

Boson	Interaction	SSB	Spin	Mass (GeV/c^2)	Color Charge	Electrical Charge (e)	Weak Charge
Gluons	Strong	No	1	0	Yes	0	No
Photon	Electro-magnetic	No	1	0	No	0	No
W^+, W^-, and Z^0	Weak	Yes	1	80.4, 80.4, and 91.2	No	+1, -1, and 0	Yes
Graviton	Gravitational	No[a]	2	0	No	0	No
X and Y	GUT (SU(5))	Yes	1	$\sim 10^{15}$	Yes	±4/3 and ±1/3	Yes
[b]	Planck-scale	[b]	[b]	[b]	[b]	[b]	[b]

[a] Assuming the graviton has zero mass.
[b] The characteristics of the Planck-scale interaction are uncertain.

The nature of the Planck-scale interaction, its mediating bosons, underlying symmetry group, families of interacting particles, energy scale, particle lifetimes, charges, SSB occurrence, and other defining characteristics are unknown. Given this uncertainty, an initial expansion of Ref.1 and the preliminary evaluation of the associated Planck-scale basic processes (i.e., vacuum bubble, closed loop, and coupled single vertex Feynman diagrams) are addressed in this paper.

5.0 Evaluation of Basic Planck Scale Feynman Diagrams

Within QFT, negative energies arise through a number of interaction processes involving virtual particles. These interactions include vacuum bubbles and closed loop interactions. Examples of a vacuum bubble and closed loop interaction are provided in Figs. 1 and 2, respectively.

Both Figs. 1 and 2 represent distinct and separate physical processes. A QED Feynman diagram and an analogous Planck-scale process are illustrated in each figure, because they have the same diagrammatic structure[1].

The conventional representation (quantities without brackets in Figs. 1 and 2) utilizes standard QED interactions involving electrons, positrons, and photons. The terms in brackets adjacent to the electrons, positrons, and photons represent particles relevant to the proposed theory and the creation of negative energies at the Planck-scale[1].

A third possible approach for the production of negative energies at the Planck-scale involves a non-standard process. The third interaction, illustrated in Fig. 3, involves two coupled single vertex diagrams. The specific nature of the coupling and evaluation of the tree level diagram are provided in subsequent discussion.

The evaluation of these diagrams is difficult because the Planck-scale interaction governing their occurrence has yet to be defined. An initial evaluation of Figs. 1 – 3 requires the specification of the interaction characteristics and sufficient parameters to

facilitate the calculation of the amplitude associated with the diagram. To facilitate the evaluation of these diagrams, the following assumptions are made:

1. Feynman rules are applicable at the Planck-scale and are appropriate tools to evaluate Figs. 1 – 3.
2. QED or an analogous QFT is applicable to evaluate the Feynman diagrams of Figs. 1 – 3.
3. Light front Quantization([18]-[20]) is applicable at the Planck-scale.
4. Light front (light cone) coordinates([19],[20]) are used to facilitate the evaluation of the amplitudes of Figs. 1 and 2.
5. Since the Planck-scale particle properties (e.g., spins and masses) are unknown, only the interaction amplitude is calculated.
6. A two-dimensional φ^4 scalar field theory[18] is assumed to be applicable to the diagram evaluation of Figs. 1 and 2. These two diagrams can be evaluated using the light front coordinates x^+ and x^- or their corresponding momenta. In the case of Figs.1 and 2, the momenta are utilized in the determination of the corresponding amplitudes.

Using these assumptions, an initial evaluation of the diagrams of Figs. 1 – 3 is performed. Although rudimentary, this effort advances the theory proposed in Ref. 1.

5.1 Light Front Coordinates and QFT

Light front coordinates([19],[20]) are useful in string theory applications. The light front coordinates x^+ and x^- are defined as two linear combinations of the time coordinate (x^0) and a chosen spatial coordinate that can be arbitrarily selected as x^1:

$$x^+ = \frac{1}{\sqrt{2}}\left(x^0 + x^1\right) \tag{3}$$

$$x^- = \frac{1}{\sqrt{2}}\left(x^0 - x^1\right) \tag{4}$$

The coordinates x^2 and x^3 are not involved in the definition of x^+ and x^-. x^+ is often described as light front time and x^- is the light front spatial distance. The coordinate x^+ and x^- axes are lines inclined 45° with respect to the x^0 and x^1 axes.

The complete set of light front coordinates are (x^+, x^-, x^2, x^3) with corresponding conjugate momenta. When utilized to evaluate the amplitudes of Figs. 1 and 2, the integrals simplify to only involve the corresponding light front conjugate momenta if a two-dimensional φ^4 scalar field theory is assumed to be applicable[18].

Ref. 18 notes that the light front form of QFT is equivalent to the usual space-like formulation. However, the mechanisms and mathematical expressions of these two formulations may be different.

5.2 Vacuum Bubbles

In the vacuum bubble diagram of Fig. 1, three virtual particles arise and subsequently return to the vacuum. The net energy output of the bubble is zero since the negative virtual energy cancels the positive virtual particle energy. Within conventional

QFT, these particles do not interact with the rest of spacetime and cannot be physically measured.

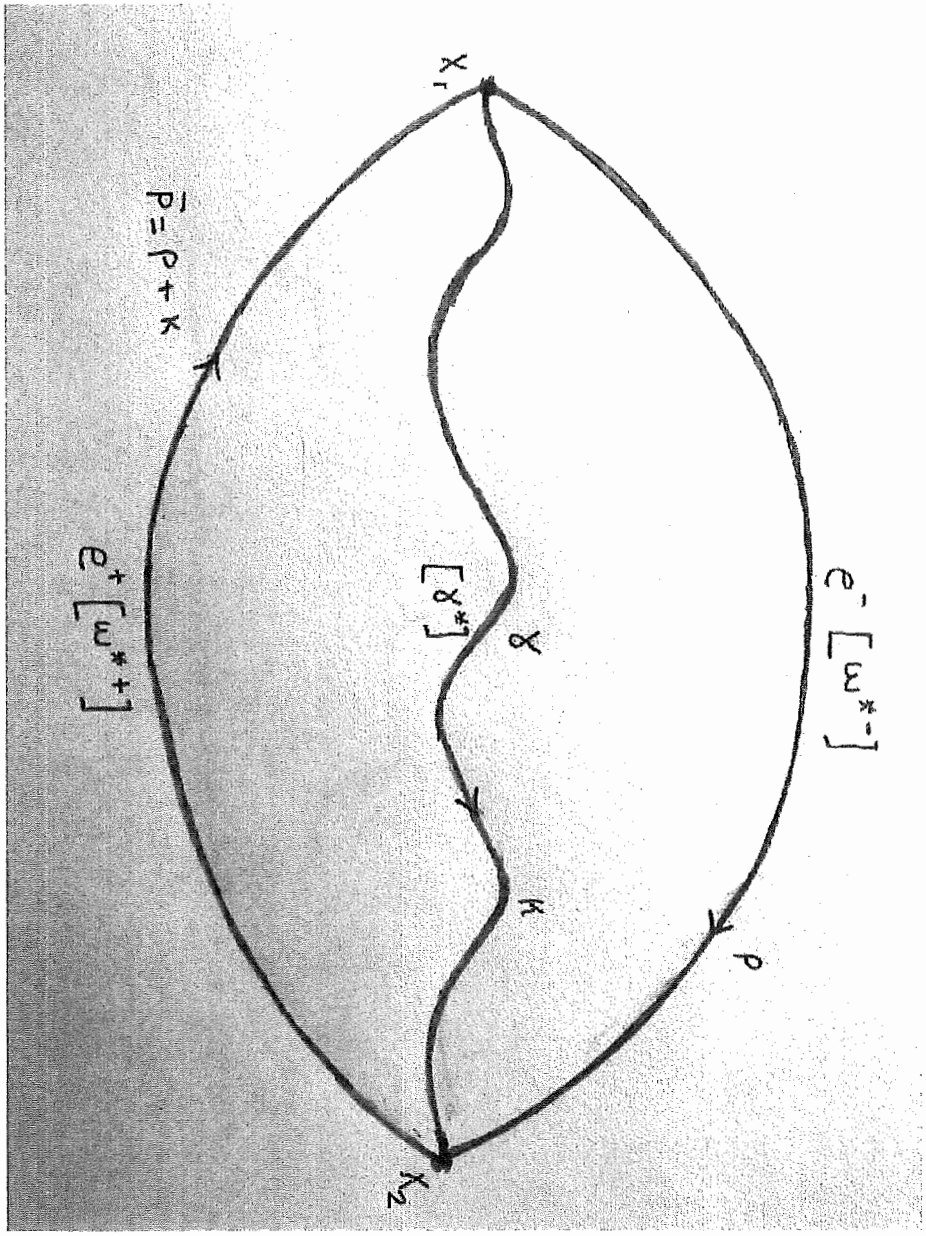


Fig. 1. Vacuum bubble Feynman diagram and the generalized Planck-scale analogue. The S_2 vacuum bubble produces negative energy particles. The terms in brackets refer to particles arising at the Planck-scale. Other quantities (electrons, positrons, and photons) refer to standard QED processes. The two processes illustrated in the figure are separate and distinct with the "*" superscript denoting Planck-scale particles.

Fig. 1 also incorporates an analogue to the QED processes that occurs at the Planck-scale. Planck-scale particles, w^{*-} analogous to the electron and w^{*+} analogous to the positron, form a diagram with γ^* as a photon analogue. The locations x_1 and x_2 specify the Planck-scale events. As a matter of notation, the superscripts *+ and *- do not indicate charge but represent the energy producing characteristic of the particle at the Planck-scale. The exact physical characteristics of the w^{*-}, w^{*+}, and γ * are to be determined in a subsequent development of the theory. Additional discussion in Fig. 1 is provided in Ref. 1.

Fig. 1 implies conservation of four-momentum at each vertex. In Fig. 1, p is the four momentum of the w^{*-}, k is the four momentum of the γ^*, and $\bar{p} = p + k$ is the four momentum of the w^{*+}. In conventional QFT, vacuum bubbles do not interact with real particles and do not alter normal spacetime. This paper assumes this conventional construct is altered at the Planck-scale.

The evaluation of vacuum bubble and closed loop diagrams with no external lines are not readily performed in an unambiguous manner [18]. One approach that can be utilized is the methodology of light front (LF) field theory. Ref. 18 notes that the diagram of Fig. 1 is not zero as often assumed. This assertion is confirmed when the evaluation is performed using LF variables[18]. In addition, more detailed LF calculations further suggest the amplitudes of Figs. 1 and 2 are non-zero([21],[22]).

The vacuum amplitude of Fig. 1 is calculated in the space-like form. Using this approach, the basic Feynman diagram rules[23] lead to the double two- dimensional integral[18] representation for the Fig. 1 amplitude

$$A(\text{Fig. 1}) = N(-i\lambda)^2 \int d^2p \int d^2(p+k) G(p) G(p+k) G(k) \qquad (5)$$

where λ is the vertex coupling constant, p and k are the light front momenta defined in Fig. 1, and G(q) and N are defined as

$$G(q) = \frac{i}{q^2 - \mu^2 + i\varepsilon} \qquad (6)$$

$$N = \frac{1}{3!} \frac{1}{i(2\pi)^4} \qquad (7)$$

In Eq. 6, μ is the scalar field particle mass, and the term 1 / 3! in Eq. 7 is a symmetry factor. The double integral in Eq. 7 can be evaluated using a number of approaches including utilizing Feynman parameters[5], an α-representation[24], and numerical methods[25]. All of these techniques yield the same result:

$$A(\text{Fig.1}) = -\frac{i\lambda^2}{\mu^2}\pi^2 NC \propto \frac{\lambda^2}{\mu^2} \qquad (8)$$

where C is a constant (~ 2.3439). The amplitude of Fig. 1 is inversely proportional to the square of the scalar field particle mass. Accordingly, the largest amplitudes occur for the lightest scalar field particles.

5.3 Closed Loops

Negative energies also occur in closed loop diagrams. For example, Fig. 2 illustrates a closed photon loop diagram and its Planck-scale analogue. In Fig. 2, p is the initial and final analogue electron (w^{*-}) four momentum, and k is the analogue photon (γ^*) four momentum.

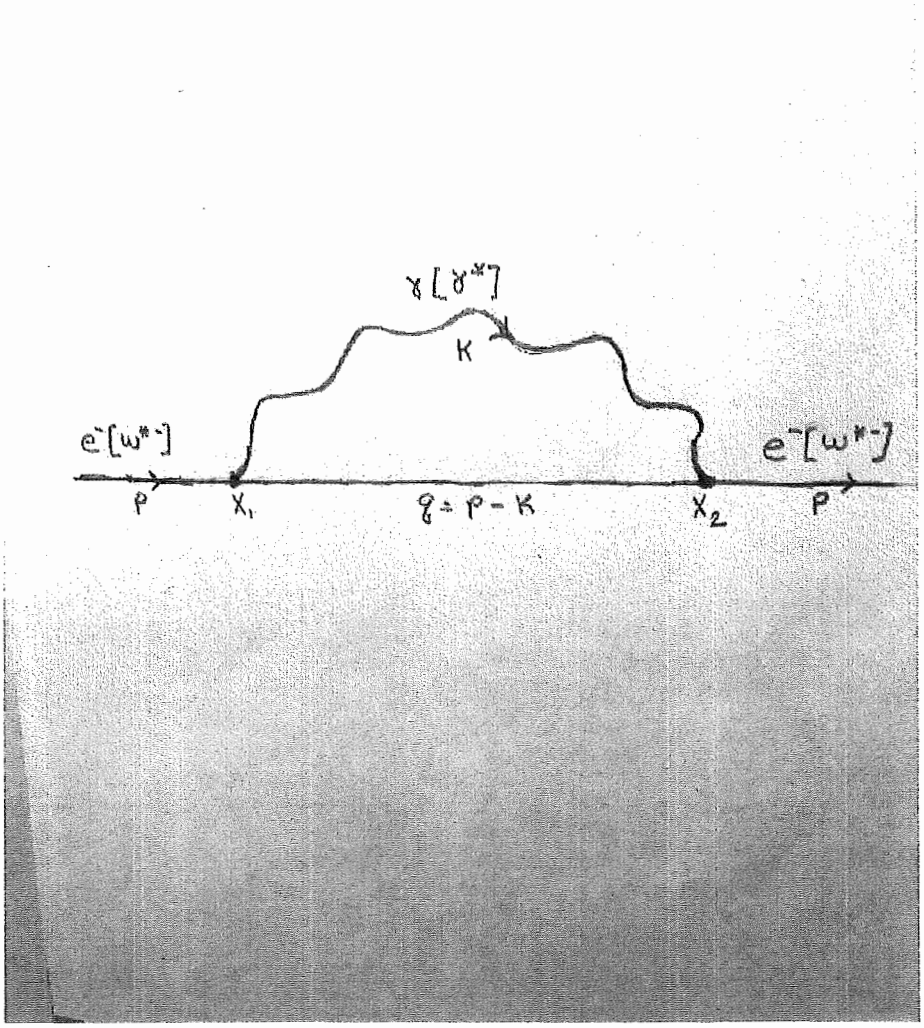


Fig. 2. Closed loop Feynman diagram and the generalized Planck-scale analogue. The S_2 loop diagram produces negative energy particles. The terms in brackets refer to particles arising at the Planck-scale. Other quantities (electrons and photons) refer to standard QED processes. The two processes illustrated in the figure are separate and distinct with the "*" superscript denoting Planck-scale particles.

Closed loop diagrams have unique characteristics. In diagrams without closed loops, all virtual particle four momenta are uniquely defined at each vertex. However, in a closed loop diagram, one of the virtual particles' four momentum can assume any value. For any given value of the virtual four-momentum k, the virtual electron four- momentum q has the value $q = p - k$. The sum of q and k must equal p, but this does not uniquely determine q and k individually.

An interesting situation arises in Fig. 2 when k has a large energy relative to the electron analogue energy (i.e., $k^0 > p^0$). In this situation, $q^0 = p^0 - k^0$ is negative which again suggests the virtual particles can have negative energies. The diagram of Fig. 2 is unique, because it has two external particle terms.

Fig. 2 also represents a single γ^* loop diagram generated by w^{*-} fields. In this generalization, the w^{*-} is analogous to the electron and γ^* would correspond to a photon analogue. As noted in the discussion of Fig. 1, the properties of the Planck-scale particles govern the particular effect that was postulated. This suggests the possibility of families of Planck-scale particles with their properties producing a particular phenomenon (e.g., black holes, wormholes, superluminal velocities, and negative gravity) outlined in Ref. 1. However, the specific characteristics of the Planck-scale particles are undefined.

Fig. 2 involves two external lines, and the corresponding Feynman amplitude is

$$A(\text{Fig. 2}) = \left(-\frac{1}{i}\right)\frac{1}{2}\frac{(-i\lambda)^2}{(2\pi)^2}\int d^2k\, G(k)G(p-k) \tag{9}$$

The approach of Ref. 18 can be used to rewrite A(Fig. 2) in terms of LF variables if the limiting condition $p = 0$ is imposed

$$A(\text{Fig. 2}) = \frac{-i\lambda^2}{16\pi^2}\int_{-\infty}^{+\infty} dk^+ \int_{-\infty}^{+\infty} dk^- \frac{1}{\left(k^+k^- - \mu^2 + i\varepsilon\right)^2} \tag{10}$$

As noted in Ref. 22, the integral over k^- requires the imposition of a cutoff Λ with the result

$$A(\text{Fig. 2}) = \left(\frac{i}{16\pi^2}\right)\lambda^2 \int_{-\infty}^{+\infty} \frac{dk^+}{k^+}\left[\frac{1}{\Lambda k^+ - \mu^2 + i\varepsilon} - \frac{1}{-\Lambda k^+ - \mu^2 + i\varepsilon}\right] \tag{11}$$

Ref. 22 notes this integral can be written in the following equivalent form

$$A(\text{Fig. 2}) = \left(\frac{i}{16\pi^2}\right)\frac{\lambda^2\Lambda}{\mu^2}\int_{-\infty}^{+\infty} dk^+ \left[\frac{1}{\Lambda k^+ - \mu^2 + i\varepsilon} - \frac{1}{\Lambda k^+ + \mu^2 - i\varepsilon}\right] \tag{12}$$

In the limit $\Lambda \rightarrow \infty$,

$$A(\text{Fig. 2}) = \left(\frac{i}{16\pi^2}\right)\frac{\lambda^2}{\mu^2}\int_{-\infty}^{+\infty} dk^+ \left[\frac{1}{k^+ + i\varepsilon} - \frac{1}{k^+ - i\varepsilon}\right] \tag{13}$$

and Eq. 13 reduces to the form[22]

$$A(\text{Fig.2}) = \left(\frac{i}{16\pi^2}\right)\frac{\lambda^2}{\mu^2}(-2\pi i)\int_{-\infty}^{+\infty} dk^+\,\delta(k^+) = \frac{1}{8\pi}\frac{\lambda^2}{\mu^2} \propto \frac{\lambda^2}{\mu^2} \qquad (14)$$

The Fig. 2 amplitude is similar to the Fig. 1 amplitude since both results are proportional to λ^2 / μ^2. This result implies that the amplitudes for Figs. 1 and 2 are governed by the coupling constant λ and the scalar field particle mass μ.

Within the restrictions of this paper, the amplitudes are maximized for the field particles with minimal mass. At this stage of development, there is no data to determine either the coupling constant or the field particle mass. However, Ref. 1 notes that these characteristics can be better defined through data involving various phenomena (e.g., black holes events, wormhole observations, superluminal velocities, and negative gravity). Excluding black hole events, none of these phenomena have been observed.

5.4 Coupled Single Vertex Diagrams

In addition to the QED analogues summarized in Figs. 1 and 2, there are possibilities beyond these conventional processes. Although there are numerous possibilities, this section addresses one specific process (i.e., the Fig. 3 tree level diagram) that is unique to the Planck-scale spacetime.

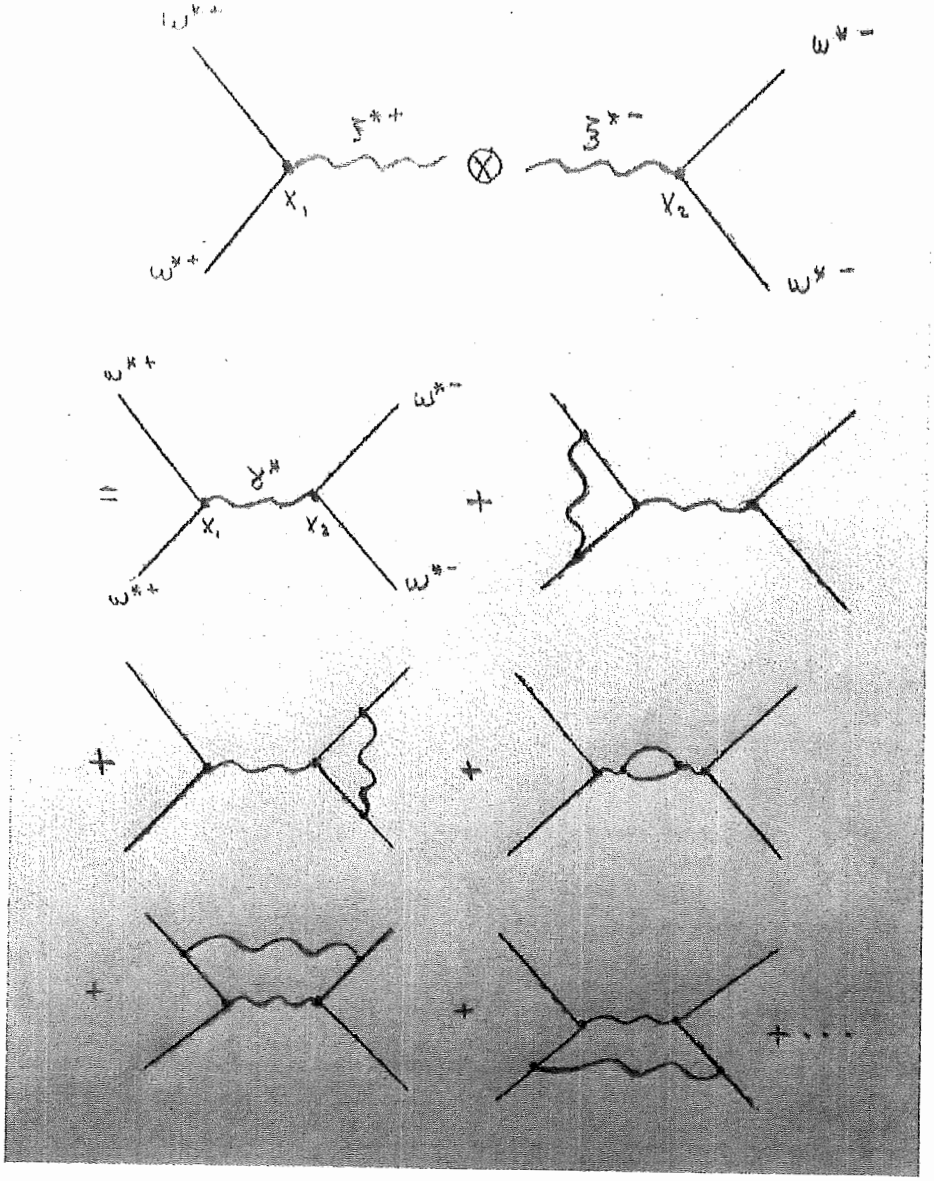


Fig. 3. Two coupled S_1 diagrams that produce positive and negative energy particles. These particles occur at the Planck-scale. The tree level and selected higher order terms are illustrated.

The additional process proposed in this paper generalizes the QFT analogue approach to negative energies, and postulates that new processes occur at the Planck-scale. These processes permit interactions to occur in a more general manner. The new process of Fig. 3 creates an additional avenue for negative energy particles production, and the creation of the postulated physical effects noted in Ref. 1.

At the current stage of development, the proposed theory is general, and not limited to specific spins of particles that interact at the Planck-scale. As such, it includes an expansion beyond normal QFT approaches. The proposed theory includes the graviton as well as known and postulated fermions and bosons and has the potential to create a theory that unifies the four fundamental forces. However, significant effort is required to develop a viable theory.

The processes of Figs. 1 and 2 are generalized to postulate ω^{*-} and ω^{*+} particles that have unspecified properties (e.g., mass and energy characteristics). In Fig. 3, these particles create a physical effect with the positive and negative energy impacts that were outlined in Ref. 1. Within the scope of Fig. 3, the γ^* particles would represent a generalized QED photon analogue that could have either positive or negative energy.

The specific characteristics of these particles including their mass, internal structure, quantum numbers, spin, parity, interaction properties, decay modes, electric dipole moment, magnetic dipole moment, and mean lifetime are unspecified. Currently, none of these characteristics are constrained. It is expected that the postulated phenomenon outlined in Ref. 3 could provide restrictions that will constrain specific values or ranges of values for these Planck-scale particles.

It is also possible to utilize other diagrams that would be excluded in QFT involving normal spacetime. Although much of the theory is in development, possible interactions involving two coupled S_1 diagrams will be illustrated.

The proposed theory presumes that the Planck-scale interactions (1) occur through analogue QFT processes (e.g., Figs. 1 and 2), (2) are created via coupled S_1 diagrams (Fig. 3), or (3) arise through other, currently unspecified processes. Although there are a variety of S_1 diagrams that could be coupled, this paper focuses on the tree level processes summarized in Fig. 3. These coupled diagrams and their associated structures are the primary focus in the remainder of this paper.

Figure 3 is a coupled diagram that has no counterpart in conventional QFT. The idea of coupled quanta is an important concept in theoretical physics with the coupling of electrons (i.e., Cooper pairs[26]) being integral to the BCS theory of superconductivity. A similar effect occurs in nuclear physics in the formulation of the pairing interaction[27], and it can be considered a nuclear analogue of the BCS theory. The QFT coupling approach noted below is also an analogue of these effects, but it is assumed to occur at the Planck-scale.

Figure 3 illustrates two separate, but coupled S_1 processes $\omega^{*-} + \omega^{*-} \rightarrow \xi^{*-}$ and $\omega^{*+} + \omega^{*+} \rightarrow \xi^{*+}$, where the ω^{*+} (ω^{*-}) are particles that excite the vacuum at the Planck-scale and subsequently create a ξ^{*+} (ξ^{*-}) particle that could have either positive or negative energy. The ω^{*+} (ω^{*-}) particles in Fig. 3 are also unspecified and could possess either positive or negative energy. An S_1 diagram represents a nonphysical process in normal spacetime, but is postulated to produce physical effects at the Planck-scale when these diagrams occur in a coupled fashion.

The production rate of the created ξ^{*+} and ξ^{*-} particles depends on the cross sections for the S_1 diagrams comprising Fig. 3. The ω^{*+} and ω^{*-} particles and associated fields could arise naturally at the Planck-scale, appear as quantum fluctuations, or be produced by external events. Natural processes creating the ω^{*+} (ω^{*-}) fields could require energies that are produced by gamma-ray bursts,
supernova events, or black hole mergers. For example, the extent and viability of any created wormholes would be determined by the intensity and duration of the ξ^{*-} fields if they possessed negative energy[1]. In a similar manner, the ξ^{*+} fields would produce strings whose episodic vibrations would lead to particle production if they possessed positive energy([28],[29]). These effects will be discussed in a subsequent paper.

Since the diagram is unique, the evaluation of Fig. 3 is not an obvious exercise. Given these considerations, an approach based on Feynman diagram[23] rules yields a specific result. This result should be treated as an initial first order estimate.

The Feynman diagram method presumes that the ξ^{*+} and ξ^{*-} annihilate and produce a photon-like analogue that connects the left and right portions of Fig. 3. This presumption leads to a basic two vertex (S_2) diagram

$$\omega^{*+}(1) + \omega^{*+}(2) \rightarrow \xi^{*+} \tag{15}$$

$$\omega^{*-}(3) + \omega^{*-}(4) \rightarrow \xi^{*-} \tag{16}$$

where the numbers 1, 2, 3, and 4 label the particles in the tree level S_2 diagram. The ξ^{*+} and ξ^{*-} combine in a manner that connects the two S_1 diagrams with the
production of a photon analogue γ^*

$$\xi^{*+} + \xi^{*-} \rightarrow \gamma^* \tag{17}$$

Given the uncertainties noted previously including the interaction, underlying symmetries, coupling constants, and particle masses, the estimate for the Fig. 3 amplitude assumes the coupling constants for the left and right portions of the diagram at their associated vertices are the same and have the value g. Given this simplification, the Fig. 3 amplitude is

$$A(\text{Fig.3}) \propto \frac{-ig^2}{(p_1 + p_2)^2 - m(\gamma^*)^2} \tag{18}$$

For the condition that $m(\gamma^*) >> (p_1 + p_2)^2$, the Fig. 3 amplitude is inversely proportional to the square of the γ^* mass that is the field particle for the Fig. 3 process.. This result is similar to the previous results for the amplitudes for Figs. 1 and 2.

$$A(\text{Fig.3}) \propto \frac{g^2}{m(\gamma)^2} \rightarrow \frac{\lambda^2}{\mu^2} \tag{19}$$

where the substitution $g \rightarrow \lambda$ and $m(\gamma) \rightarrow \mu$ is used to facilitate comparison to the Figs. 1 and 2 amplitude results. For all three diagrams, the largest amplitudes are attributed to the smallest field particle masses.

The aforementioned method of evaluating Fig. 3 is not unique. Other approaches could be utilized since the coupling of the left and right portions of Fig. 3 has not been

uniquely defined. Other approaches could incorporate the angular momentum coupling of the left and right portions of the tree level diagram, coupling of the ξ^{*+} and ξ^{*-} using methods other than that proposed in Ref. 1 including creation of multiparticle states, other Planck-scale gauge bosons, and photon analogues that interfere in a variety of ways.

Table 2 summarizes the assumptions and methods used to evaluate the three Planck-scale amplitudes of Figs. 1 – 3. Given the assumptions and methods limitations noted in Table 2, all three diagram evaluations suggest the amplitudes are proportional to λ^2 / μ^2.

Table 2

Summary of Assumptions and Methods Used to Evaluate Planck-Scale Amplitudes

Process	Assumptions and Methods	Amplitude
Vacuum Bubble (Fig. 1)	1. Basic Feynman rules are applicable	$\propto \frac{\lambda^2}{\mu^2}$
	2. Two-dimensional φ^4 scalar field theory is applicable	
	3. Light front coordinates are utilized to facilitate the amplitude calculation	
	4. QED or an analogous QFT is applicable	
Closed Loop (Fig. 2)	1. Basic Feynman rules are applicable	$\propto \frac{\lambda^2}{\mu^2}$
	2. Two-dimensional φ^4 scalar field theory is applicable	
	3. Light front coordinates are utilized to facilitate the amplitude calculation	
	4. QED or an analogous QFT is applicable	
	5. Evaluated with the cutoff limit $\Lambda \rightarrow \infty$,	
	6. Evaluated with the $p = 0$ limitation	
Coupled Single Vertex Diagram (Fig. 3)	1. Basic Feynman rules are applicable	$\propto \frac{\lambda^2}{\mu^2}$
	2. QED or an analogous QFT is applicable	
	3. The $m(\gamma^*) >> (p_1 + p_2)^2$ assumption is similar to the Fig. 1 $p = 0$ limitation	
	4. ξ^{*+} and ξ^{*-} annihilate to produce a photon analogue γ^+	

6.0 Conclusions

The methodology of this paper extends the theory of Ref. 1 and outlines an additional approach to evaluate the Feynman diagrams of Figs. 1, 2, and 3. Although the results of this paper are relatively simplistic, they do provide an initial quantification of the amplitudes of the associated Planck-scale processes. For all three diagrams, the amplitudes are proportional to the square of the coupling constant and inversely proportional to the square of the field particle mass.

References

[1] J. J. Bevelacqua, Possible Planck-scale physical production mechanisms and consequences of negative energies—Initial formulation, Physics Essays **34** (3) (2021), 342-351.
[2] S. Tomonaga, On a Relativistically Invariant Formulation of the Quantum Theory of Wave Fields, Prog. Theor. Phys. **1** (1946), 27-42.
[3] J. Schwinger, Quantum Electrodynamics. I. A Covariant Formulation, Phys. Rev. **74**, (1948), 1439-1461.
[4] R. P. Feynman, The Theory of Positrons, Phys. Rev. **76** (1949), 749-759.
[5] R. P. Feynman, Space-Time Approach to Quantum Electrodynamics, Phys. Rev. **76** (1949), 769-789.
[6] S. Glashow, Partial-symmetries of weak interactions, Nucl. Phys. **22** (1961), 579-588.
[7] A. Salam and J. C. Ward, Weak and electromagnetic interactions, Nuovo Cimento. **11** (1959), 568-577.
[8] S Weinberg, A Model of Leptons, Phys. Rev. Lett. **19** (1967) 1264-1266.
[9] D. J. Gross and F. Wilczek, Ultraviolet Behavior of Non-Abelian Gauge Theories, Phys. Rev. Lett. **30** (1973), 1343-1346.
[10] F. Wilczek, Quantum field theory, Rev. Mod. Phys. **71** (1999), S85-S95.
[11] D. J. Gross, Nobel Lecture: The discovery of asymptotic freedom and the emergence of QCD, Rev. Mod. Phys. 77 (2005), 837-849.
[12] P. W. Higgs, Broken Symmetries, Massless Particles and Gauge Fields, Phys. Lett. **12** (1964), 132-133.
[13] P. W. Higgs, Broken Symmetries and the Masses of Gauge Bosons, Phys. Rev. Lett. **13** (1964), 508-509.
[14] Y. Nambu, Axial Vector Current Conservation in Weak Interactions Phys. Rev. Lett. **4** (1960), 380-382.
[15] J. Goldstone, Field Theories with Superconductor Solutions, Nuovo Cimento **19**, (1961), 154-164.
[16] T. W. B. Kibble, Symmetry Breaking in Non-Abelian Gauge Theories Phys. Rev. **155** (1967), 1554-1561.
[17] K. Grotz and H. V. Klapdor, The Weak Interaction in Nuclear, Particle and Astrophysics, Bristol, Hilger (1990).
[18] L. Martinovič and A. Dorokhov, Vacuum loops in light-front field theory, Phys. Lett. B **811** (2020), 135925-1 – 135925-6.
[19] M. Burkardt, Light front quantization of the sine-Gordon model, Phys. Rev. D **47** (1993), 4628–4633.
[20] V. A. Franke, Y. V. Novozhilov, S. A. Paston, and E. V. Prokhvatilov, in Quantum Theory in Honour of Vladimir A. Fock, Part 1, UNESCO, St. Petersburg University, Euro-Asian Physical Society, p. 38-97 (1998).
[21] S. –J. Chang and S. –K. Ma, Feynman Rules and Quantum Electrodynamics at Infinite Momentum, Phys. Rev. **180** (1969), 1506-1513.
[22] T. –M. Yan, Quantum Field Theories in the Infinite-Momentum Frame, IV. Scattering Matrix of Vector and Dirac Fields and Perturbation Theory, Rev. D 7 (1973), 1780-1800.

[23] M. E. Peshkin and D. V. Schroeder, An Introduction to Quantum Field Theory, Boulder, CO, Westview Press (1995).
[24] V. A. Smirnov, Analytical Tools for Feynman Integals, Springer Tracts in Modern Physics, Springer-Verlag (2012).
[25] A. J. Davydychev and J. R. Tausk, Two loop selfenergy diagrams with different masses and the momentum expansion, Nucl. Phys. B **397** (1993), 123-142.
[26] L. N. Cooper, Bound Electron Pairs in a Degenerate Fermi Gas, Phys. Rev. **104** (1956), 1189-1190.
[27] J. Blomqvist and S. Wahlborn, Shell model calculations in the lead region with a diffuse nuclear potential, Ark. Fys. **16** (1959), 545-566.
[28] D. Matsunami, L. Pogosian, A. Saurabh, and T. Vachaspati, Decay of Cosmic String Loops due to Particle Radiation Decay of Cosmic String Loops due to Particle Radiation, Phys. Rev. Lett. **122** (2019), 201301-1 – 201301-5.
[29] J. J. Bevelacqua, Nonrelativistic Open String Model Parameters Based on Particle Mass and Lifetime Values, J. Nucl. Particle Phys. **10**, (2020), 1-8.

HADRONIC JOURNAL 45, 253 - 262 (2022)
DOI 10.29083/HJ.45.03.2022/SC253

MASS FORMULA OF HADRONS FOR THREE GENERATION QUARK STANDARD MODEL AND DIFFERENCES ON ELECTROMAGNETIC MASS OF BARYON-MESONS

Yi-Fang Chang
Department of Physics
Yunnan University
Kunming, 650091, China
yifangch@sina.com

Received June 1, 2022

Abstract

First, we research some further possible directions of experiments. Second, based on the known mass and lifetime formulas, three generation hadrons are discussed. Third, we study differences on the electromagnetic mass of baryons and mesons, and their results are calculated for some hadrons.

Keywords: particle physics; hadron; mass; formula; electromagnetic interaction.

Copyright © 2022 by Hadronic Press, Inc., Palm Harbor, FL 34684, USA

I. Introduction

Now the three generations of quark-leptons and Higgs bosons have been confirmed. The further directions should be to find: 1. various new particles composed of the Standard Model. Different generation quarks have a certain periodicity, in which most notable are $\eta_c = c\bar{c}$ and $\Upsilon = b\bar{b}$. 2. Find out that whether gluons have three generations, or are only the excited gluons of known first generation. For example, the existence of three generations of W-Z, or three generations are all W-Z excited states. 3. Are there three generations of Higgs particles?

It is known that electromagnetic interactions violate the isospin invariance, and cause the splitting of the isospin multiples and mass differences of proton and neutron, etc [1]. The model for the electromagnetic mass splitting and the hyperfine mass structure of isospin multiples is discussed [2,3]. In this paper, combining the standard quark model, we research three generation hadrons and various differences on the electromagnetic mass of baryons and mesons.

2. Known Mass and Lifetime Formulas

First generation quark-lepton is very precise, which includes the anomalous magnetic moments of electron by the renormalization theory, and Lamb shift. Second and third generations (quark-lepton) must probably be renormalization first, which is concrete renormalization, rather than theoretical renormalization.

This may correspond to the Rosen-Ross mass formula of leptons-meson [4]:

$$M = m_e(1+\frac{n}{2\alpha}). \qquad (1)$$

When n=3, M=206.554 m_e =105.55MeV $\approx$ $m(\mu^{\pm})$ (105.66); n=4, M=275.072 m_e=140.56MeV $\approx$ $m(\pi^{\pm})$(139.57).

The octet of hadrons has different S and I, while the quark mass is very uncertain. The GMO mass formula corresponds to diatomic molecules with vibration-rotation, and corresponds to ud-s two bodies. We proposed a modified GMO mass formula [5,4,6]:

$$M = M_0 - AS + B[I(I+1) - S^2/2]. \qquad (2)$$

Such not only baryons and mesons are unified M=m, and Eq.(2) agrees accurately with the experiments. For the $J^p = 1^+/2$ baryon octet, let $M_0 = 908$, A=228 and B=40MeV, so

$$m(N) = 938, m(\Lambda) = 1116, m(\Sigma) = 1196, m(\Xi) = 1314\,\text{MeV}. \quad (3)$$

For the $J^{pc}=0^{-+}$ meson octet, let *A*=0, M_0 =549.4MeV and $B=-207.22$ MeV, so

$$m(\pi^0)=134.96, m(K^0)=497.6, m(\eta)=549.4\,\text{MeV}. \quad (4)$$

The neutral mesons agree completely within the range of error, and M=m.

Assume that hadrons are formed from the emergence string. Usual string should possess two moving states: oscillation and rotation, so we proposed corresponding potential and the equation of the emergence string, whose energy spectrum is namely the GMO mass formula and its modified accurate mass formula (2) [6]. Based on the Y-Q and I-U symmetries between mass and lifetime on the general SU(3) theory, we derived the lifetime formulas of hyperons and mesons:

$$\tau = A[2U(U+1)-Q/2], \quad (5)$$

and $$\tau = A'[(1/2)+2U(U+1)-Q/2-Q^2/3]. \quad (6)$$

They agree better with experiments [4,6]. It is a new method on lifetime of hadrons described by quantum numbers. They are symmetrical with the corresponding mass formulas, and can be unified for mass and lifetime. Further, these formulas may extend to describe masses and lifetime of heavy flavor hadrons [6].

According to the symmetry of s-c quarks in the same generation, the heavy flavor hadrons which made of u,d and c quarks may be classified by SU(3) octet and decuplet. Eq.(2) can be developed to ud-c, and assume that rotation I is the same and vibrations S and C are different. We derived the symmetrical mass formula [6]:

$$M = M_0 + AC + B[I(I+1)-C^2/2]. \quad (7)$$

Let M_0 =875, A=1453, B=84MeV, so m(N)=938, $m(\Lambda_c^+)=2286$, $m(\Sigma_c)=2454$ MeV, and we predicted $m(\Xi_{cc})=3673$MeV and so on [6]. In July 2017 LHC observed a new and very charming baryon $\Xi_{cc}^{++}=ucc$, whose mass is 3621*MeV* [7]. It agrees more on formula (7), whose error only is 1.4%. Further, we predict m(Ω_{cc}^+)=3946, or 3950.8, or 3908.2*MeV*, etc. Moreover, this method may be extended to other heavy flavor hadrons, and predict $m(\Xi_{bb})$=10396.8 or 10348.9*MeV*, etc. It is a quantitative and testable theory [8].

Based on the modified accurate mass formula (7) and applied to heavy flavor hadrons, we predicted $m(\Xi_{bb})$ =10396.8 or 10348.9*MeV*, etc. We proposed various lifetime formulas of heavy flavor hadrons, which very agree with experiments. This can be unified for mass and lifetime. We discussed an

approximate simplified supersymmetry theory based on the known symmetrical particles and their excited states [9].

3. Three Generations of Hadrons

For general mass formula

$$M = M_0 + AC + BI(I+1) + DC^2 . \quad (8)$$

It cannot agree completely. But, assume that

$$M = M_0 + AC + BI(I+1) + DIC . \quad (9)$$

Let M_0=638, A=1650, B=400, D=-620MeV, so

m(N)=938, $m(\Lambda_c^+)$=2288, $m(\Sigma_c)$=2468, and $m(\Xi_{cc})$=3618MeV. (10)

If let M_0=638, A=1642, B=400, D=-(3/2)B=-600MeV, so

m(N)=938, $m(\Lambda_c^+)$=2280, $m(\Sigma_c)$=2480, and $m(\Xi_{cc})$=3622MeV. (11)

Both the Planck blackbody radiation formula and the Bohr orbit are based on the oscillators. In the mass formula (2) S corresponds to vibration, and I corresponds to rotation. There is only rotation between u-d, S and C are different vibrations, they are basically symmetrical, A=228, and 1453; and s and c are I=0, and no rotation. Further, S-C-B-T are all vibration-only, and correspond to $an + bn^2 + cn^3 + ... + kn^k$.

Mass ($K^+ = u\bar{s}$):($D^+ = \bar{d}c$):($B^+ = u\bar{b}$)=494:1870:5279=3.785:2.823. The difference ratio of the excited states is smaller and smaller.

Mass ($\Lambda : \Lambda_c^+ : \Lambda_b^0$)=1116:2286:5623=2.048[=($\Sigma_c^0 : \Sigma^-$)=2453:1197=2.049]: 2.458. The difference ratio of the excited states is bigger and bigger.

Mass (Ξ_c^+=usc: $\Xi^0 = uss$)=2468:1315=1.877; ($\Omega_c^0 : \Omega^-$)=2695:1672=1.612. The ratio of the excited states along with increasing mass is smaller and smaller.

Three generation quarks are (d,u)(s,c)(b.t) in the Standard Model. They include confirmed hadrons and yet undiscovered hadrons.

For mesons first generation $\pi^+ = u\bar{d}$, $\pi^0 = d\bar{d}$, $\eta = u\bar{u}$.

Second generation $K^+ = u\bar{s}$, $K^0 = d\bar{s}$ (S=1), and $\rho = s\bar{s}$.

$D^+ = \bar{d}c$, $D^0 = \bar{u}c$ (C=1), and $D_s^+ = c\bar{s}$, $\eta_c = c\bar{c}$.

Third generation $B^+ = u\bar{b}$, $B^0 = d\bar{b}$ (B=1), and $B_c^+ = c\bar{b}$, $B_s^0 = s\bar{b}$, $\Upsilon = b\bar{b}$.

$u\bar{t}, d\bar{t}, s\bar{t}, c\bar{t}$ (T=1), and $b\bar{t}, t\bar{t}$.

For baryons first generation p=uud, n=udd (J=1/2), and Δ^{++} =uuu, $\Delta^- = ddd$ (J=3/2).

Second generation $(\Lambda, \Sigma^0) = uds$, $\Sigma^+ = uus$, $\Sigma^- = dds$ and $\Xi^0 = uss$, $\Xi^- = dss$ (J=1/2); Ω^-=sss (J=3/2).

(Λ_c^+, Σ_c^+)=udc, Σ_c^{++} =uuc, Σ_c^0 =ddc, and Ξ_{cc}^{++} =ucc, Ξ_{cc}^+ =dcc, $\Omega_{cc}^+ = scc$, Ω_{ccc}^{++} =ccc, and Ξ_c^+ =usc, Ξ_c^0 =dsc, Ω_c^0 =ssc.

Third generation (Λ_b^0, Σ_b^0)=udb, Σ_b^+ =uub, Σ_b^- =ddb (I=0,1), and Ξ_b^0 =usb, Ξ_b^- =dsb, ucb, dcb (I=1/2) and ubb, dbb, sbb, cbb, bbb and Ω_b^- [6315.64, 6330.30, 6339.71, 6349.88MeV]=ssb, scb, ccb

udt, uut, ddt (I=0,1) and ust, dst; uct, dct (I=1/2) and utt, dtt, stt, ctt, ttt and sst, sct, cct and ubt, dbt, sbt, cbt, bbt, btt.

It found $\Omega_b^- \to \Xi_b^0 K^-$, Γ [Ω_b^- (6316)]<2.8MeV, Γ [Ω_b^- (6330)]<3.1MeV, Γ [Ω_b^- (6340)]< 1.5MeV, Γ [Ω_b^- (6350)]=1.4MeV [10].

It is known that K interactions are weaker than π. If it is generalized, then C, D, B and T interactions will be weaker and weaker. Further, the three-generation interactions should become weaker and weaker.

If the interaction is required to be the same, all three generation quark-lepton exchange particles must be zero mass.

Baryons p, n, Λ_c^+ (I=0), Σ_c (I=1) and Ξ_{cc} (I=1/2) form a side of SU(4). If three sides are mass-equidistant, m(Ω^-)=1672.5, m(Ω_c^0)=2695.2, the mass difference 1022.7, so that m(Ω_{cc}^+)=3717.9, m(Ω_{ccc}^{++})=4740.6 (J=3/2). But, m(Δ^{++})=1232, m(Σ_c^{++})=2520, mass-equidistant m(Ξ_{cc}^{++})=3808, m(Ω_{ccc}^{++})=5096 (J=3/2). m(Δ^-)=1232, m(Σ_c^0)=2520, mass-equidistant m(Ξ_{cc}^+)=3808, m(Ω_{ccc}^{++})=5096 (J=3/2). Both m(Ω_{ccc}^{++}) are different, so it can be determined that the mass-equidistant is merely approximated.

4. Electromagnetic Mass Difference of Hadrons

We proposed the topological structures of particles which determine different spins, and a trefoil knot of left hand and a trefoil knot of right hand (Fig.1) is possibly similar to two spins s=±1/2 of three quarks (nucleons p and n, etc). Basic mesons of two quark form a line, and s=0.

Further, this model is extended to general fermions and bosons. It may contribute to resolve the spin crisis. Based on the topological structures of particles, we researched possible violation of Pauli exclusion principle (PEP),

and proposed some predictions. Moreover, we discussed other effects of topological structure, and some applications [11].

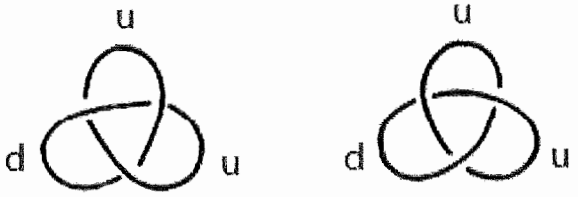


Fig.1 A trefoil knot of right hand and a trefoil knot of left hand

One-dimensional p(u-d-u) and n(d-u-d), three quarks in plane correspond to triangles, but the two angles of u-u are mutually exclusive, the charges at both ends of proton are 2/3, and the repulsion is larger; and the charges at both ends of neutron are -1/3, and the repulsion is smaller.

The both angles are different, and the magnetic moments are also different. This is probably related to the electromagnetic mass difference. Σ^+ (u-s-u) is similar to p, but Σ^-(d-d-s) is mutually repulsion, and mass is the biggest. The electromagnetic mass difference of hadrons should originate from electromagnetic interaction in hadrons. Santilli reviewed and upgraded the isorepresentation of the spin 1/2 of nucleons according to the isotopic branch of hadronic mechanics, and is consistent with the spin J=1 and the magnetic moment of the Deuteron with the numeric value λ=2.65557 of Bohm hidden variable [12].

The Coulomb force is:

$$F_{em} = -Q_1 Q_2 / r^2 , \qquad (12)$$

and corresponding potential is:

$$V_{em} = -Q_1 Q_2 / r . \qquad (13)$$

We calculate simply the known electromagnetic mass difference of some hadrons [13].

1) For nucleon n(udd)-p(uud)=ud(d-u)=1.29MeV. Here ud(e=1/3)-d(-1/3) is attraction, and ud(e=1/3)-u(2/3) is repulsion.

$$F_{em} = [-(1/9) + (2/9)]e^2 / r^2 = (1/9)e^2 / r^2 \text{ is repulsion. (14)}$$

Such $V = me^2 / r = m\hbar c / 137r .$ (15)

V(m) is bigger, so the distance r is smaller.

$r = 197MeVfm/137\times 9\times 1.29MeV = 0.124fm = 1.24\times 10^{-14}$cm. (16)

It is the distance of ud-d(u) in n(p).

2) $m(\Sigma^0 = uds) - m(\Sigma^+ = uus) = us(d-u)$ =3.27MeV. Here us(e=1/3)-d(-1/3) is attraction, and us(e=1/3)-u(2/3) is repulsion.

$F_{em} = [-(1/9)+(2/9)]e^2/r^2 = (1/9)e^2/r^2$ is repulsion. (17)

r=(1.598/3.27)$\times 10^{-14}$cm=0.489$\times 10^{-14}$cm. (18)

It is the distance of us-d(u).

3) $m(\Sigma^- = dds) - m(\Sigma^0 = uds) = ds(d-u)$ =4.81MeV. Here ds(e=-2/3)-d(-1/3) is repulsion, ds(e=-2/3)-u(2/3) is attraction.

$F_{em} = [(2/9)-(4/9)]e^2/r^2 = -(2/9)e^2/r^2$ is attraction. (19)

r=2(1.598/4.81)$\times 10^{-14}$cm=0.664$\times 10^{-14}$cm. (20)

This is 0.5 of (16), and is the distance of ds-d(u).

4) $m(\Xi^- = dss) - m(\Xi^0 = uss) = ss(d-u)$ =6.48MeV. Here ss(e=-2/3)-d(-1/3) is repulsion, ss(e=-2/3)-u(2/3) is attraction.

$F_{em} = [(2/9)-(4/9)]e^2/r^2 = -(2/9)e^2/r^2$ is attraction. (21)

r=2(1.598/6.48)$\times 10^{-14}$cm=0.493$\times 10^{-14}$cm. (22)

It is the same with (18), and is the distance of ss-d(u).

m(d-u)=1.29, 3.27, 4.81, 6.48MeV are different from one another. The first two are equal repulsion, and us(d-u)-ud(d-u)=(d-u)u(s-d)=1.98. The later two are 2 times attraction, and ss(d-u)-ds(d-u)=(d-u)s(s-d)=1.67.

For J=3/2, $m(\Sigma^0 = uds) - m(\Sigma^+ = uus) = us(d-u)$ =0.9MeV.

$m(\Sigma^- = dds) - m(\Sigma^0 = uds) = ds(d-u)$ =3.5MeV=

$m(\Xi^- = dss) - m(\Xi^0 = uss) = ss(d-u)$ =3.2MeV.

5) Mesons $m(K^0 = d\bar{s}) - m(K^+ = u\bar{s}) = \bar{s}(d-u)$ =3.97MeV. Here $\bar{s}$ (e=1/3)-d(-1/3) is attraction, $\bar{s}$ (e=1/3)-u(2/3) is repulsion.

$F_{em} = [-(1/9)+(2/9)]e^2/r^2 = (1/9)e^2/r^2$ is repulsion. (23)

r=(1.598/3.97)$\times 10^{-14}$cm=0.403$\times 10^{-14}$cm. (24)

It is the distance of $\bar{s}$-d(u).

6) Heavy flavor mesons $m(D^+ = c\bar{d}) - m(D^0 = c\bar{u}) = c(\bar{d}-\bar{u})$ =4.78MeV, in which c(e=2/3)-$\bar{d}$ (1/3) is repulsion, c(e=2/3)-$\bar{u}$ (-2/3) is attraction.

$F_{em} = [(2/9)-(4/9)]e^2/r^2 = -(2/9)e^2/r^2$ is attraction. (25)

r=2(1.598/4.78)$\times 10^{-14}$cm=0.669$\times 10^{-14}$cm.=(20). (26)

It is the distance of c-d(u).

7) Heavy flavor mesons $m(B^0 = d\bar{b}) - m(B^+ = u\bar{b}) = \bar{b}(d-u)$ =0.33MeV, in which $\bar{b}$ (e=1/3)-d(-1/3) is attraction, $\bar{b}$ (e=1/3)-u(2/3) is repulsion.

$$F_{em} = [-(1/9) + (2/9)]e^2/r^2 = (1/9)e^2/r^2 \text{ is repulsion.} \quad (27)$$

$$r=(1.598/0.33)\times 10^{-14}\text{cm}=4.842\times 10^{-14}\text{cm}. \quad (28)$$

It is the distance of $\bar{b}$ -d(u).

8) Heavy flavor hyperons $m(\Xi_c^0 = dsc) - m(\Xi_c^+ = usc) = sc(d-u)$ =4.0MeV, in which sc(e=1/3)-d(-1/3) is attraction, sc(e=1/3)-u(2/3) is repulsion.

$$F_{em} = [-(1/9) + (2/9)]e^2/r^2 = (1/9)e^2/r^2 \text{ is repulsion.} \quad (29)$$

$$r=(1.598/4.0)\times 10^{-14}\text{cm}=0.400\times 10^{-14}\text{cm}. =(24). \quad (30)$$

It is the distance of sc-d(u).

9) $m(\Xi_b^- = dsb) - m(\Xi_b^0 = usb) = sb(d-u)$ =5.1MeV [14], in which sb(e=-2/3)-d(-1/3) is repulsion, sb(e=-2/3)-u(2/3) is attraction.

$$F_{em} = [(2/9) - (4/9)]e^2/r^2 = -(2/9)e^2/r^2 \text{ is attraction.} \quad (31)$$

$$r=2(1.598/5.1)\times 10^{-14}\text{cm}=0.627\times 10^{-14}\text{cm}. =(20). \quad (32)$$

It is the distance of sb-d(u).

In conclusion, the different electromagnetic fields between the two parts in the hadrons must be repulsion:

$$F_{em} = [-(1/9) + (2/9)]e^2/r^2 = (1/9)e^2/r^2, \quad (33)$$

or attraction:

$$F_{em} = [(2/9) - (4/9)]e^2/r^2 = -(2/9)e^2/r^2. \quad (34)$$

The complexity is that there is interaction, but it is asymptotically free and there should be no interaction, so the above model can be approximately applied. The attraction derives in a large electromagnetic mass difference. The electromagnetic mass difference is inversely proportional to the quark distance. For attractions 3) 6) 9) are basically the same. For repulsion 2) 4) 8) are basically the same.

$$[m(\Xi^-) - m(\Xi^0)]/2=3.24\text{MeV} \approx m(\Sigma^0) - m(\Sigma^+)=3.27\text{MeV}. \quad (35)$$

This is just 2 times. 1.29+3.27=4.56=4.81 is also 2 times. For repulsion (24) and (30) are the same.

Moreover, there have various relations of above masses: 3(1.29)=3.87, and 4(1.29)=5.16. But, [(s-d)+(d-u)]/2=(1.98+1.29)/2=1.64=s-d=1.67. 1.29+6.48=7.77=3.27+4.81=8.08.

In a word, attractions can be classified as three 4.81 and 6.48. Repulsions can be classified as two 4, and 3.27, 1.29, 033, which are more complex. But, the relation between these and the quark mass is not obvious. The resonance mass is very uncertain.

Further, we predict hyperons

$m(\Xi_{cc}^{+}=dcc)-m(\Xi_{cc}^{++}=ucc)=cc(d-u)$ =4.0MeV. Such $m(\Xi_{cc}^{+})$ =3625MeV.

$m(\Sigma_c^{+}=udc)-m(\Sigma_c^{++}=uuc)=uc(d-u)=-0.9$ MeV probably exist error.

We proposed the differences on the electromagnetic mass of baryons and mesons [4]:

$$\Delta m=Q[d_1(1.5-0.5Q-S)-d_2]=(1.5d_1-d_2)Q-d_1QS-0.5d_1Q^2. \quad (36)$$

Let d_1=1.8MeV, $d_2=m_e$=0.51MeV,

$$\Delta m=1.80n-0.51. \quad (37)$$

This is the Chong-Shou Gao relation of differences on the electromagnetic mass of baryons. Further, it may develop to heavy flavor hadrons.

Table 1. Differences on the electromagnetic mass of $J^P=(1/2)^+$ baryons

	Calc. Value (37)	Obs. Value [13] MeV
m(n)-m(p)	1.29	1.29
m(Σ^0)- m(Σ^+)	3.09	3.27
m(Σ^-)- m(Σ^0)	4.89	4.81
m(Ξ^-)- m(Ξ^0)	6.69	6.48

This can be attributed to Heisenberg earliest idea: the mass difference of n-p is caused by electromagnetism, and we combine the quark model.

References

[1] W. Heisenberg, *Introduction to the Unified Field Theory of Elementary Particles*. London: Interscience Publishers. **1966**.

[2] H. Harari, *Phys.Rev.Lett.* 17(**1966**):1303; 18(**1967**):319.

[3] B.T. Feld, *Models of Elementary Particles*. Blaisdell Publishing Company. **1969**.

[4] Yi-Fang Chang, *New Research of Particle Physics and Relativity*. Yunnan Science and Technology Press. **1989**; *Phys.Abst.* 93(**1990**)No1371.

[5] Yi-Fang Chang, *Hadronic J.* 7(**1984**):1118.

[6] Yi-Fang Chang, *International Review of Physics*. 6(**2012**):261.

[7] R. Aaij, B. Adeva, M. Adinolfi, et al. (LHCb Collaboration), *Phys. Rev. Lett.* 119(**2017**):112001.

[8] Yi-Fang Chang, *Hadronic J.* 41(**2018**):221.

[9] Yi-Fang Chang, *Hadronic J.* 41(**2018**):335.

[10] R. Aaij, et al. (LHCb Collaboration), *Phys. Rev. Lett.* 124(**2020**):082002.
[11] Yi-Fang Chang, *Hadronic J.* 44(**2021**):311.
[12] R. M. Santilli, *Progress in Physics*. 18(**2022**):74.
[13] M. Tanabashi, K. Hagiwara, K. Hikasa, et al. Particle Data Group. *Phys.Rev*. D98(**2018**): 030001.
[14] R. Aaij, et al. (LHCb Collaboration), *Phys. Rev. Lett.* 128(**2022**):162001.

HADRONIC JOURNAL 45, 263 - 295 (2022)
DOI 10.29083/HJ.45.03.2022/SC263

NEGENTROPIC THERMODYNAMICS I: PROOF OF THE NEGENTROPIC NATURE OF THE TRANSFORMATION OF HEAT INTO WORK

José C. Íñiguez
Retired Instructor of Science and Mathematics
1227 21st Street
Douglas, AZ 85607
iniguez.jose@gmail.com

Received May 28, 2022

Abstract

The negentropic nature (negative total-entropy change) of the energy-upgrading process represented by the transformation of heat into work is here evinced through a thermodynamic argument centered on the identity of effects existing between an irreversible heat transfer and the concatenation of a reversible heat engine with a frictional step in which the engine's work-output is degraded into heat at the temperature of its cold reservoir. The said argument shows that negating the negentropic nature of the heat-to-work transformation leads to a negation of the state-function nature of the entropy. The negentropic nature of this transformation contradicts the law of increasing entropy, the so-called supreme law of nature, and provides the theoretical foundation of what we have called negentropic or dialectical thermodynamics.

Keywords:

Reversible engines, entropy, negentropy, the flaws in Clausius' thermodynamics, the flaws in post-Clausius' thermodynamics, proof for the negentropic nature of the transformation of heat into work.

Copyright © 2022 by Hadronic Press, Inc., Palm Harbor, FL 34684, USA

1. Introduction

According to Planck:

> The significance of the second law of thermodynamics depends on the fact that it supplies a necessary and far-reaching criterion as to whether a definite process which occurs in nature is reversible or irreversible…A process which can in no way be completely reversed is termed irreversible, all other processes reversible. That a process may be irreversible, it is not sufficient that it cannot be directly reversed…The full requirement is, that it be impossible, even with the assistance of all agents in nature, to restore everywhere the exact initial state when the process has once taken place… *Every physical or chemical process in nature takes place in such a way as to increase the sum of the entropies of all the bodies taking any part in the process. In the limit, i.e., for reversible processes, the sum of the entropies remains unchanged.* This is the most general statement of the second law of thermodynamics. (1990, pp. 86-87, 84, 103)

This "...sum of the entropies of all the bodies taking any part in the process..." is to be here referred as the 'total-entropy change'.

2. Antecedents: Reversible heat engines

The essential elements for a theoretical analysis of a reversible heat engine, such as that represented in Figure1(a), are the following:

1. A hot reservoir of temperature T_h
2. A cold reservoir of temperature T_c, $T_h > T_c$
3. A working substance or variable body which for the purpose of this discussion will be taken to be an ideal gas, and
4. A mechanical reservoir (*MR*) acting as depository (source) of the work produced (consumed) in the operation.

One cycle in the operation of this engine finds description in terms of the following concatenation of reversible changes experienced by the working substance:

1) *AB*: An isothermal expansion at the temperature of the hot reservoir. Here the amount of heat Q_h taken in by the gas from the hot reservoir is transformed into an equivalent amount of work W_h that ends up in the mechanical reservoir.

2) *BC*: An adiabatic expansion lowering the temperature of the gas from T_h to T_c

3) *CD*: An isothermal compression at the temperature of the cold reservoir. Here an amount of work W_c retrieved from the mechanical reservoir is used to carry on this compression. The dissipated work, in the form of the equivalent amount of heat Q_c, ends up in the cold reservoir; and finally, closing the cycle

4) *DA*: An adiabatic compression raising the temperature of the gas from T_c to T_h

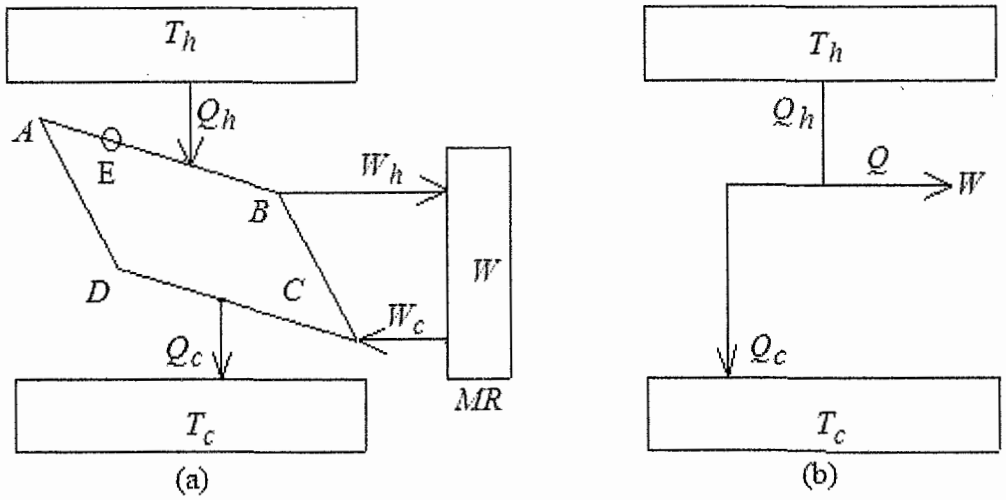


Figure 1. Two representations of a reversible heat engine. The cyclical evolution of the ideal gas defined in (*a*) by the closed concatenation of processes *AB-BC-CD-DA* produces, out of the amount of heat Q_h taken in by the gas from the hot reservoir, the two final effects represented in (*b*), these being the reversible transfer of the amount of heat Q_c from the hot to the cold reservoir, and the reversible transformation of the amount of heat Q into an equivalent amount of work W. In the figures it is understood that $Q_h = Q_c + W$, $Q = W$, and $W = W_h - W_c$. Point E is used below in the argument directed to the identification and evaluation of the total-entropy changes of the two transformations depicted in (*b*).

A particular characteristic of these reversible cycles is represented by the inverse nature of isentropic processes *BC* and *DA*. By this we mean that the effects of any such process are precisely reverted or canceled by the other one. That this is so starts by recognizing that for adiabatic processes the first law reduces to $\Delta U = -W$. For the cycle of Figure $1(a)$, with an ideal gas as working substance, the following expressions can be written: $\Delta U_{BC} = -W_{BC} = C_v(T_c - T_h)$ and $\Delta U_{DA} = -W_{DA} = C_v(T_h - T_c)$, where C_v stands for the constant-volume heat capacity of the gas. The addition of the previous expressions produces $\Delta U_{BC} + \Delta U_{DA} = 0$ and $W_{BC} + W_{DA} = 0$. These results bring to light the facts, on the one hand, that the negative internal energy change and associated temperature decrease experienced by the gas along *BC,* are offset by the positive internal energy change and associated temperature increase associated to *DA*; and on the other, that an amount of work identical to that produced along expansion *BC* is expended along compression *DA*. Being this the case, it then follows that the work effect (W) of the cycle is the sole matter of isothermal and reversible processes *AB* and *CD* in the following form $W = W_h - W_c$, where, as noted, W_h stands for the work produced by expansion *AB,* and W_c the one consumed along compression *CD*.

Recognition of the fact that the two effects produced by one cycle in the operation of a reversible heat engine relate to the way the engine's input heat is split into two streams with different destinations, allowed Clausius to forward the following alternative and equivalent description to the one just given for this engine:

> In a simple cyclical process two variations in respect to heat take place, viz. that a certain quantity of heat is converted into work (or generated out of work), and another quantity of heat passes from a hotter into a colder body (or vice versa)....For the former of these two variations we have already employed the word 'transformation,' inasmuch as we said, when work was expended and heat thereby produced, or conversely when heat was expended and work thereby produced, that the one has been 'transformed' into the other. We may use the word 'transformation' to express the second variation also (which consists in the passage of heat from one body into another, which may be colder or hotter than the first), inasmuch as we may say that heat of one temperature 'transforms' itself into heat of another temperature. On this principle we may describe the result of a simple

> cyclical process in the following terms: Two transformations are produced, a transformation from heat into work (or vice versa) and a transformation from heat of a higher temperature to heat of a lower (or vice versa). The relation between these two transformations is therefore that which is to be expressed by the second Main Principle (Clausius, 1879, pp. 91-92)

Clausius' heat-engines-based work on the second law has as distinctive characteristic the fact that it centers its attention on the total-entropy changes of his so-called transformations i.e., on the combined entropy changes of the totality of bodies involved in each of them, rather than in the individual entropy changes of these bodies. As a matter of fact, and in accordance with the concluding statement of his last quote, Clausius' first statements of the second law were advanced in terms of this notion, e.g. "The algebraic sum of all the transformations which occur in a cyclical process must always be positive or in the limit equal to zero" (1879, p. 213). Once the transformations or effects produced by a heat engine (or any other thermodynamic process, for that matter) have been recognized, then the entropy changes of the bodies involved in each of these transformations can be properly combined to produce the transformations' respective total-entropy changes. This way, the total-entropy change for a heat engine will be determined by the combined total-entropy changes of its constitutive transformations. The transit from the individual entropy changes of the bodies involved in a transformation to the entropy change of the collective brings, as will be evinced by the following arguments, economy to the discussion and facility to the mathematical manipulations.

The two reversible (*rev*) transformations associated to engine $1(a)$ will be from now on represented, as they appear from left to right in Figure $1(b)$, by the following self-evident notation: $[Q_c(T_h) \rightarrow Q_c(T_c)]_{rev}$ and $[Q(T_h) \rightarrow W]_{rev}$.

It is not unlikely for certain transformations to be identical to some identifiable thermodynamic process or processes. Such is the case with the transformation of heat into work taking place in a reversible engine which turns out to be identical with an isothermal and reversible expansion. The use of the term 'transformation' will be eventually extended to irreversible transfers of heat, as well as the irreversible transformation of work into heat.

3. Clausius and the total-entropy changes for his transformations

Chapter IV of Clausius' book "The Mechanical Theory of Heat" (1879), describes in detail the way he ascertained the 'equivalence-values' or total-entropy changes for the two transformations produced by one cycle in the operation of a reversible heat engine.

These values, given below in equations (1) and (2)

$$\Delta S_{total,Clausius}[Q_c(T_h) \rightarrow Q_c(T_c)]_{rev} = -\frac{Q_c}{T_h} + \frac{Q_c}{T_c} \qquad (1)$$

and

$$\Delta S_{total,Clausius}[Q(T_h) \rightarrow W]_{rev} = -\frac{Q}{T_h} \qquad (2)$$

allowed him to show that a constant total-entropy evolution was the distinctive characteristic of reversible engines, such as engine 1(a) described in Figure 1:

$$\begin{aligned}\Delta S_{total,Clausius}[\text{heat engine}]_{rev} &= \Delta S_{total,Clausius}[Q_c(T_h) \rightarrow Q_c(T_c)]_{rev} \\ &+ \Delta S_{total,Clausius}[Q(T_h) \rightarrow W]_{rev} = -\frac{Q_c}{T_h} + \frac{Q_c}{T_c} - \frac{Q}{T_h} \\ &= -\frac{Q_h}{T_h} + \frac{Q_c}{T_c} = 0 \qquad (3)\end{aligned}$$

The total-entropy changes for the transformations produced by one cycle in the operation of a reversible refrigerator, namely the inverse cycle of reversible engine 1(a), can, on reason of the state function nature of the entropy (Clausius, 1879, p. 90) in combination with the fact that the coupling or concatenation of any given reversible process with its inverse restores the initial condition, be written in the following form:

$$\Delta S_{total,Clausius}[Q_c(T_c) \rightarrow Q_c(T_h)]_{rev} = -\frac{Q_c}{T_c} + \frac{Q_c}{T_h} \qquad (4)$$

and

$$\Delta S_{total,Clausius}[W \rightarrow Q(T_h)]_{rev} = \frac{Q}{T_h} \qquad (5)$$

These last two equations combine, in the way indicated below, to define the also zero total-entropy change for a reversible refrigerator:

$$\Delta S_{total,Clausius}[\text{refrigerator}]_{rev} = \Delta S_{total,Clausius}[Q_c(T_c) \rightarrow Q_c(T_h)]_{rev} + \Delta S_{total,Clausius}[W \rightarrow Q(T_h)]_{rev} = -\frac{Q_c}{T_c} + \frac{Q_c}{T_h} + \frac{Q}{T_h} = \frac{Q_h}{T_h} - \frac{Q_c}{T_c} = 0 \qquad (6)$$

Equations (1) through (6) form the backbone of Clausius work on the second law. This assertion stems from the fact that one of his first statements of the second law was centered on them:

> Hence the second Main principle of the Mechanical Theory of Heat, which in this form may perhaps be called the principle of the Equivalence of Transformations, can be expressed in the following terms:
>
> If we call two transformations which may cancel each other without requiring any other permanent change to take place, Equivalent Transformations, then the generation out of work of the quantity of heat Q of temperature (T) has the equivalence value Q/T; and the transference of the quantity of heat Q from temperature T_1 to temperature T_2 has the Equivalence-value $Q\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$ ... (1879, pp. 100-101)

In order to complement the previous statement, let us add that $-Q/T$ should be used when "...the quantity of heat Q has been transformed not out of work into heat, but out of heat into work." (1879, p. 98)

With specific reference to reversible cyclical processes Clausius stated:

> If the cyclical process under consideration is reversible, then, however complicated it may be, it can be proved hat the transformations which occur in it must cancel each other so that their algebraical sum is equal to nothing." (1879, p. 102)

Clausius extension of his work from reversible to irreversible cyclical processes led him to conclude that:

> The algebraic sum of all the transformations occurring in a circular process can only be positive, or, as an extreme case, equal to nothing." (1862, p. 84)

Eventually, he was able extend his results to all thermodynamic processes:

> The algebraic sum of all the transformations occurring in any alteration of condition whatever can only be positive, or, as an extreme case, equal to nothing." (1862, p. 211)

This last statement, it should be recognized, is nothing more than the law of increasing entropy worded in terms of the combined value of the transformations. In terms of entropy, it would read: The total-entropy change for reversible processes is zero, and positive for all others.

As the following discussion evinces, Clausius work proved to be, however, permeated with inconsistencies and contradictions.

For the law of increasing entropy, the ultimate discriminator between reversible and irreversible processes is the total-entropy change.

In the words of Planck:

> The second law of thermodynamics states that there exists in nature for each system of bodies a quantity which, by all changes of the system either remains constant (in reversible processes) or increases in value (in irreversible processes). This quantity is called, following Clausius, the entropy of the system. (Planck, 1990, p. 88)

That this discriminating role ascribed by Planck to the total-entropy change is absent from Clausius work finds confirmation in the identity in total-entropy changes assigned by him to the reversible and the irreversible transfers of an amount of heat Q between temperatures T_1 and T_2, as well as to the reversible and irreversible transformations of a certain amount of work into an equivalent amount of heat of a certain temperature. In Clausius quote below, uncompensated means irreversible.

> In many cases the magnitude of the Uncompensated Transformation is obtained directly from the equivalence value of the transformations...If for example a quantity of heat Q is generated by any process such as friction, and this is finally imparted to a body of temperature T, the uncompensated transformation thus produced has the value Q/T. Again, if a quantity of heat Q has passed by conduction from a body of temperature T_1 to another of temperature T_2 then the

uncompensated transformation is $Q\left(\frac{1}{T_2}-\frac{1}{T_1}\right)$. (Clausius, 1879, p. 214)

That he is assigning the same total-entropy change to the reversible and irreversible versions of the same process is certainly not our interpretation of the previous quote. No such interpretation is needed when he is declaring precisely that in the first three lines of said quote. The combination of these total-entropy changes, with equations (1) and (5), allows us to realize that for Clausius, in contradiction to his own law of increasing entropy, it was true that:

$$\Delta S_{total,Clausius}[Q_c(T_h) \rightarrow Q_c(T_c)]_{rev} = \Delta S_{total,Clausius}[Q_c(T_h) \rightarrow Q_c(T_c)]_{irr} = -\frac{Q_c}{T_h}+\frac{Q_c}{T_c} \quad (7)$$

And

$$\Delta S_{total,Clausius}[W \rightarrow Q(T_h)]_{rev} = \Delta S_{total,Clausius}[W \rightarrow Q(T_h)]_{irr} = \frac{Q}{T_h} \quad (8)$$

Additional discussions on the contradictions subsumed by Clausius work on the second law can be found at (Íñiguez 2021a).

Given the nature of the flaws in Clausius analysis the only course of action in accord with what is understood as rigorous scientific praxis would have been to subject the whole of his work on the second law to a profound revision, exposing the results of this effort to the analysis/criticism of all knowledgeable or interested in the matter, and accepting whatever conclusion was obtained, even if in contradiction with the law of increasing entropy. This, however, didn't happen. Apparently, and I am here speculating, the idiosyncrasies of the leading thermodynamicists of the day prevailed over good science and the decision was made to declare this law valid beyond doubt. As clearly expressed in that famous quote of Eddington, doubting this law, subjecting it to a rational analysis, or exposing ideas in opposition to it became discouraged, forbidden: "But if your theory is against the second law of thermodynamics I can give you no hope; there is nothing for it but to collapse in deepest humiliation." (1929, p. 74).

Assuring the uninterrupted rule of the law of increasing entropy called first of all for a surreptitious corrective action capable of providing the said law

with a self-consistent, flawless, and credible foundation. One that would resist any kind of questioning or inquiry. The fact that this law is, still to this day, considered as holding "...the supreme position among the laws of Nature." (Eddington, p.74) can be taken as evidence of the quality of the said corrective actions; but also, as a measure of the gullibility of thermodynamicists (Íñiguez, 2021b)

The present author has no factual information as to the who or the when of the changes introduced in Clausius work. This is a matter for historians of thermodynamics, a discipline which in the words of McGlashan "...is in fact a much more difficult subject than thermodynamics itself and much less well understood." (1966, p. 226) What this author has, is factual evidence that Clausius work experienced a change in what was called above 'the backbone of his work'. While the total-entropy changes for his transformations given in equations (1), (2), (4), and (5) transited suddenly and unexplainably from the non-zero values shown there, to zero; those for the reversible engine and its associated refrigerator retained, on the other hand, the zero values originally given to them by Clausius, as shown in equations (3) and (6). A selective corrective action, indeed. It is this change in the values of the transformations what embodies the transition from Clausius' thermodynamics to what this author has called post-Clausius thermodynamics (*PCT*) (Íñiguez 2021b).

The how this was accomplished is the matter of the following section. The arguments there presented will unveil the fact that the *source* of validity of the law of increasing entropy which in Clausius work was a (flawed) thermodynamic analysis, became, in post-Clausius' thermodynamics, an argument of authority, a *fiat*.

The fact that apparently no record exists in the published thermodynamic literature of the changes experienced by Clausius work, as previously described, is the reason why this modification was qualified above as surreptitious

4. The total-entropy changes for the two transformations taking place in a reversible engine according to current thermodynamic wisdom or post-Clausius' thermodynamics.

In terms of the transformations produced by one cycle in its operation, and in accord with the zero total-entropy change prescribed by the law of increasing

entropy, the total-entropy change for a reversible heat engine, such as engine 1(a) described above, can be written as follows:

$$\begin{aligned} &\Delta S_{total,PCT}[\text{heat engine}]_{rev} \\ &= \Delta S_{total,PCT}[Q_c(T_h) \rightarrow Q_c(T_c)]_{rev} \\ &+ \Delta S_{total,PCT}[Q(T_h) \rightarrow W]_{rev} = 0 \end{aligned} \qquad (9)$$

The identification of the process or processes responsible for the changes associated to each of these transformations, as well as of the total-entropy change which in accord with post-Clausius thermodynamics should be ascribed to each of them, is the objective of the following discussion.

Let us start recognizing that in the engine depicted in Figure 1(*a*), as in any other heat engine, it is true that $Q_h > Q_c$, i.e., the amount of heat taken in by the ideal gas from the hot reservoir along *AB* is larger than the amount of heat it gives out to the cold reservoir along *CD*. This fact allows us to realize that there exists some point *E* on isotherm *AB* that defines a transit *EB* in which the amount of heat received by the gas from the hot reservoir turns out to be identical to the amount of heat Q_c given out by the gas to the cold reservoir along *CD*. The approximate location of point *E* is shown in Figure 1(a). Assuming that the operation of the cycle is to start at point *E*, consideration will be given to the effect of the concatenation of processes *EB-BC-CD-DA*. Actually, on reason of the fact that the effects of adiabatic and reversible process *BC* are precisely cancelled by adiabatic and reversible process *DA*, the effect of the said concatenation reduces to that produced by the combination of isothermal and reversible processes *EB* and *CD*. Along *EB* the hot reservoir transfers to the ideal gas an amount of heat Q_c. The ideal gas, on its part, transforms this heat into an equivalent amount of work W_c which is subsequently spent in carrying on compression *CD*, with the spent work, in the form of its equivalent amount of heat Q_c, ending up in the cold reservoir.

The fact that having left the hot reservoir of temperature T_h to be transformed into W_c along *EB*, Q_c reappears entering the cold reservoir due to the dissipation of W_c along *CD*, allows us to realize that the only effect produced by this concatenation of processes is that of the reversible transfer of the amount of heat Q_c from the hot to the cold reservoir

Recognition of the facts that processes *BC* and *DA* are by definition isentropic, and that the total-entropy changes for isothermal and reversible

expansion *EB* and isothermal and reversible compression *CD*, according to post-Clausius' thermodynamics are both zero, leads us to:

$$\begin{aligned} & \Delta S_{total,PCT}[Q_c(T_h) \rightarrow Q_c(T_c)]_{rev} \\ &= \Delta S_{total,PCT}\{[Q_c(T_h) \rightarrow W_c] + [W_c \rightarrow Q_c(T_c)]\}_{rev} \\ &= \Delta S_{total,PCT}[EB]_{rev} + \Delta S_{total,PCT}[CD]_{rev} = 0 + 0 \\ &= 0 \qquad (10) \end{aligned}$$

The explanation forwarded by Klotz and Rosenberg (1986, p. 133), similar to any other explanation offered by *PCT* for the zero total-entropy change of isothermal and reversible volume changes of ideal gases goes as follows "If Q is the heat absorbed by the system, then $-Q$ must be the heat absorbed by the surroundings...Hence, for the system plus surroundings $\Delta S_{total} = \Delta S_{sys} + \Delta S_{surr} = 0$."

An identical result to that given by equation (10), obtained via similar procedures, can be found in *PCT's* literature (Dickerson, 1969, pp. 162-163; Moore, 1972, 88-89).

Let us now agree that the process *AE* needed to bring the cycle to its conclusion is an isothermal and reversible expansion at the temperature of the hot reservoir. Through it the ideal gas absorbs an amount of heat Q from the hot reservoir and transforms it into an equivalent amount of work W. Since no other effect but this can be associated to this expansion, it follows that it is *AE,* an isothermal and reversible expansion, the one responsible for the net-work output of the cycle, or in other words, for bringing forward the transformation of Q into W. The fact that the total-entropy change for this isothermal and reversible expansion is, in accord with the rationale of current thermodynamic wisdom, exemplified above via the quote of Klotz and Rosenberg, also zero, allows us to write:

$$\Delta S_{total,PCT}[Q(T_h) \rightarrow W)]_{rev} = \Delta S_{total,PCT}[AE] = 0 \qquad (11)$$

The reversible label attached to both of the previous transformation stems from the fact that the initial condition is restored when the previous operation, ending at *E*, is concatenated with the inverse cycle *EA-AD-DC-CB-BE.* In it, the work W available in the mechanical reservoir is retrieved from it and used to drive isothermal compression *EA*, with the spent work, in the form of an equivalent amount of heat Q, returning back to the hot reservoir where it originally came from. The remaining of the cycle is in charge of the transference of Q_c back to the hot reservoir: Having been taken in by the gas

from the cold reservoir along expansion *DC,* it will be released by it to the hot reservoir along compression *BE.*

The total-entropy change for reversible heat engine 1(*a*) will be now calculated via the addition of equations (10) and (11), as follows:

$$\Delta S_{total,PCT}[\text{heat engine}]_{rev} = \Delta S_{total,PCT}[Q_c(T_h) \rightarrow Q_c(T_c)]_{rev} + \Delta S_{total,PCT}[Q(T_h) \rightarrow W]_{rev} = 0 + 0 = 0 \qquad (12)$$

Not only does the combination of the total-entropy changes given by equations (10) and (11) confirms the prescription of the law of increasing entropy (*LIE)* stated by eq. (9), they also individually comply with the constant total-entropy reversibility condition, a must, taking into account that these are total-entropy changes.

Note here that while in Clausius' thermodynamics the zero total-entropy change for a reversible engine, as evinced by eq. (3), comes as a result of the addition of two non-zero quantities, in post-Clausius' thermodynamics, as evinced by eq. (12), the very same result is obtained via the addition of two zeros.

Even if at first sight the previous arguments might look free of inconsistencies, contradictions, flaws, etc., the fact is that they are incorrect for the simple reason that they have taken as true a notion that is far from being that. Identifying this notion and exposing the logical imbroglio associated to it, is the matter of the next section.

5. post-Clausius thermodynamics incorrect conceptualization of isothermal and reversible volume changes of ideal gases.

If a closer look is taken at the arguments producing the zero-total entropy changes for the transformations shown in equations (10) and (11), it will be realized that the keystone of those arguments –the notion that made them possible– is the one asserting the zero total-entropy changes for isothermal and reversible volume changes of ideal gases. The problem with this procedure is that no thermodynamic proof exists to support the zero-total entropy change for an isothermal and reversible ideal-gas expansion (*IRE*), or compression (*IRC*). Most, if not all, of those reading these lines would say that no such proof is needed; that its validity rests on the validity of the law of increasing entropy which dictates the constant total-entropy nature of all reversible processes. This is an understandable position, but incorrect

nonetheless, given the fact that it is not the law of increasing entropy the one guaranteeing the zero-total entropy change of *IRE*'s and *IRC*'s but the other way around. To understand how and why this is so, is the matter of the following discussions.

Let us start considering the conductive transfer of an element of heat dQ from a heat reservoir of temperature T to another heat reservoir of the same temperature. The fact that these two bodies are the only ones taking part in this reversible process allows writing its total-entropy change in the following manner:

$$dS_{total}[dQ(T) \rightarrow dQ(T)] = (-dQ/T) + (dQ/T) = 0 \qquad (13)$$

The self-evident expression appearing inside the left hand-side bracket of the previous equation serves to identify the process whose total-entropy change is being evaluated. Read from left to right, the two terms on the right-hand side stand, respectively, for the entropy changes of the heat-releasing and heat-receiving reservoirs. The fact that the entropy change produced by the previous equation is the result of the combination of the entropy changes of all the bodies involved in the process being considered, is what makes it a total-entropy change. Its equality to zero indicates that this system composed of two identical temperatures heat reservoirs is a system in equilibrium and as such, unable to output any work.

Let us now replace one of the heat reservoirs of the previous discussion with a diathermal-walled cylinder which appropriately fitted with a weightless and frictionless piston serves as vessel to one mole of an ideal gas at the same temperature of the remaining reservoir. The piston has been appropriately coupled to a, say, weight-and-pulley mechanical reservoir. Let us now assume that an element of heat dQ is transferred from the reservoir to the gas in order to be transformed, via its isothermal and reversible expansion, into an equivalent amount of work dW. It should be remembered here that on reason of being solely dependent on the temperature, the internal energy of an ideal gas remains constant in an isothermal process. Under this condition, the first law transits from $dU = dQ - dW$, to $dQ = dW$ which evinces the quantitative transformation from heat into work taking place in these reversible processes (or from work into heat in their reversible inverses). The work dW produced by this reversible expansion out of the heat received from the reservoir appears in the surroundings as a proportional increase in the potential energy (height) of the weight.

In regard to this process, it shouldn't be difficult to understand that the gas needs to be in possession of dQ before it can actually start converting it into work along its expansion. What this means is that the transfer of dQ from the reservoir to the gas precedes the transformation of this heat into dW. This simple insight leads to the realization that the transference of heat from the reservoir to the gas, and the transformation of this heat into work by the expansion of the gas are two different processes, the former preceding the latter. If so, the total-entropy change for this expansion must be written as follows:

$$dS_{total}[IRE] = dS[\text{heat res.}] + dS[\text{gas}] + dS[dQ(T) \rightarrow dW] \qquad (14)$$

Or, as follows:

$$dS_{total}\,[IRE] = dS[dQ(T) \rightarrow dQ(T)] + dS[dQ(T) \rightarrow dW] \qquad (15)$$

In acronym *IRE*, the *R* stands for reversible, therefore $dS_{total}\,[IRE]$ refers to the total-entropy change of a reversible process, and if so, also reversible are the entropy changes appearing in the right-hand side of this, or similar equations. In any other situation the '*rev*' subindex will be added to prevent ambiguities.

The fact that the combination of the first two terms on the right-hand side of equation (14) define the entropy change for the transference of dQ from the reservoir to the gas, explains its re-expression as $dS[dQ(T) \rightarrow dQ(T)]$ in eq. (15). In both of the previous equations $dS[dQ(T) \rightarrow dW]$ represents the actual transformation of heat into work produced by the expansion of the gas. In accord with eq. (13), the following re-expression of eq. (15) can be written:

$$dS_{total}[IRE] = -dQ/T + dQ/T + dS[dQ(T) \rightarrow dW] \qquad (16)$$

The first two entropy changes on the right-hand side of the previous equation correspond, from left to right, to the heat-releasing reservoir, and to the heat-receiving gas. After simplification, equation (16) becomes:

$$dS_{total}\,[IRE] = dS[dQ(T) \rightarrow dW]_{rev} \qquad (17)$$

The previous equation makes evident the fact that, in opposition to Klotz and Rosenberg assertion previously noted, no statement can be made about the total-entropy change of an isothermal and reversible ideal-gas expansion without having previously ascertained the entropy change associated to the transformation of heat into work. The fact that no such *knowledge* exists

makes it impossible to make a statement about $dS_{total}\,[IRE]$ and, consequently, puts on hold the validity of what has been called "...the greatest scientific achievement of the nineteenth century" (I.P. Bazarov, wikiquote), namely the law of increasing entropy. Let me be perfectly clear in this regard, if $dS[dQ(T) \rightarrow dW]_{rev}$ is *proven from first principles* to be different from zero, then the validity of the law of increasing entropy would be disproved because in such a case, just like a counterexample disproves a universal, the isothermal and reversible expansion to which eq. (17) makes reference, in opposition to the prescription of the law of increasing entropy, would be shown taking place with a non-zero total-entropy change.

But if no proof exists for $dS[dQ(T) \rightarrow dW]_{rev} = 0$, how is it that post-Clausius thermodynamics asserts the zero-total entropy change of this transformation, and in doing so, via the arguments leading to equations (10) and (11), provide support for the law of increasing entropy via the zero-total entropy changes of reversible cyclical processes?

The answer is: By *fiat*!

Barrow explains it as follows:

> We are at liberty, you should recognize, to ascribe any features to this new entropy function that we like, the requirement being that we construct a function that is self-consistent and allows us to form a useful expression of the second law. In this vein we further specify that, for all processes $dS_{mech\,res} = 0$. (1973, p. 168)

The subindex '*mech res*' appearing in the previous quote, short for 'mechanical reservoir' originates in another conceptual fiasco of post-Clausius' thermodynamics as in it the weight in the mechanical reservoir is the embodiment of the transformation of heat into work, in such a way that it is the entropy change of the perpetually-in-equilibrium internal state of this mechanical body the one defining the total-entropy change of the thermodynamic process through which heat ends up being transformed into work (Bent, Chapters 9, 12) A detailed discussion on this issue forms part of previous paper by this author (Íñiguez, 2021b)

The previous discussion reminds the present author of that statement from Angrist and Hepler which reads: "Thermodynamics is too important to be left to the thermodynamicists." (1967, p. v)

The *fiat* making

$$dS_{total,PCT}[dQ(T) \rightarrow dW]_{rev} = 0 \qquad (18)$$

Implies, necessarily, the zero total-entropy change of the processes in charge of producing this transformation, namely isothermal and reversible ideal-gas expansions, for what else is a thermodynamic process but its effects. Therefore

$$dS_{total,PCT}[dQ(T) \rightarrow dW]_{rev} = dS_{total,PCT}[IRE] = 0 \qquad (19)$$

The restoration of the initial condition produced by the coupling of any reversible process with its inverse, combined with the state function nature of the entropy function leads, necessarily, to the following equation for isothermal and reversible ideal-gas compressions [*IRC*]:

$$dS_{total,PCT}[dW \rightarrow dQ(T)]_{rev} = dS_{total,PCT}[IRC] = 0 \qquad (20)$$

The assertions found in current thermodynamics textbooks about the zero total-entropy change for *IRE*'s and *IRC*'s are not backed, as it is usually believed, by the law of increasing entropy (*LIE*) but by the *fiat* so eloquently expressed by Barrow's quote. Without equation (19) no statement about the total-entropy change for reversible cycles would be possible, and if so, neither a universal statement about the total-entropy changes of reversible processes.

The American saying 'there is no such thing as a free lunch' finds eloquent support in the conceptual confusion that the said *fiat* brought with it.

The effect of substituting the *fiat* given by equation (18) in the equation for the total-entropy change of an isothermal and reversible ideal-gas expansion, namely equation (16), has the effect of making the latter identical with equation (13), which is the one defining the total-entropy change of the reversible transfer of the amount of heat dQ between two heat reservoirs of temperature T. In short, the substitution of eq. (18) in eq, (16) produces:

$$dS_{total,PCT}[IRE] = dS[dQ(T) \rightarrow dQ(T)] = -dQ/T + dQ/T \qquad (21)$$

The problem with this result is that it erases the distinction existing between the heat-transfer process and the isothermal and reversible expansion. While the former –the transfer of dQ between two reservoirs of identical temperature T – is a process in equilibrium and as such unable to output any work, the *IRE* is, on its part, a process in its way to equilibrium, the fact that it can output work is the proof of this.

> From a system in equilibrium no work can be obtained, but a system on the way to equilibrium may be made to yield useful work. (Maron and Prutton, 1965, p. 189)

Accepting post-Clausius' thermodynamics *fiat* decreeing the zero-total entropy change for isothermal and reversible ideal-gas expansions demands from us, against the most elementary principle of logic, namely the principle of contradiction, further acceptance of the notion that a system can be out of equilibrium and in equilibrium at the same time (Norton, 2016, p. 43). The mere fact that an *IRE* outputs work makes it a non-equilibrium process and as such it cannot, should not be assigned a zero total-entropy change.

When we 'lose' a certain amount of work into heat we also lose all those capabilities that make the former a superior-quality energy form than heat, namely change-inducing, structure-forming, and chaos-ordering, among others. In simpler terms, associated to this transformation of work into heat is the loss on our part of the capacity to constructively transform our surroundings. A simple comparison between the things you can accomplish with an electric motor and those you can accomplish with a torch or a bonfire will bring meaning to our previous assertions. It is to this loss of capabilities that the positive entropy change associated to the transit $dW \rightarrow dQ$ makes reference to. In the words of Weber and Meissner:

> The entropy change of the system plus surroundings may be viewed a quantitative measure, or index, of the degradation of energy as work to energy as heat. (1957, p. 168)

Recognition of the fact that the transit in the opposite direction, namely $dQ \rightarrow dW$ is responsible for an increase in our transformational capabilities should be reason enough to assign the transformation of heat into work with a negative entropy change. If quality loss (energy degrading) is entropic, quality gain (energy upgrading) should be negentropic; or as Heraclitus recognized some 2500 years ago "The way up is the way back" (Haxton, 2003, p. 69).

It needs to be pointed out that if post-Clausius' thermodynamics chose the *fiat*-path to solve the *Gordian knot* represented by eq. (17) it was on reason of its inability to provide a proof to the effect that $dS_{total}[dQ(T) \rightarrow dW]_{rev} = 0$. The reason for this is simple. No such proof exists. If anything can be proved about $dS_{total}[dQ(T) \rightarrow dW]_{rev}$, as will be done below, is precisely the opposite, namely that

$dS_{total}[dQ(T) \rightarrow dW]_{rev} \neq 0$. Before we do that, we will take a closer look at the total-entropy change of irreversible transfers of heat.

6. An irreversible transference of heat

Figure 2 depicts the irreversible transfer of an amount of heat Q_h from a hot reservoir of temperature T_h, to a cold reservoir of temperature T_c. The process is irreversible because the only way possible to restore the initial condition without additional changes setting in, namely the spontaneous –unaided- transit of Q_h back from the cold to the hot reservoir, is denied by experience.

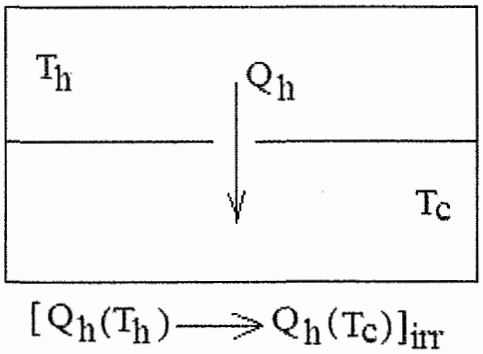


Figure 2. Depiction of the conductive, irreversible transfer of an amount of heat Q_h from a hot reservoir of temperature T_h to a cold reservoir of temperature T_c. This process finds identification via the self-evident transformation notation there included in which the sub-index '*irr*' makes reference to its irreversible nature.

The fact made evident by Figure 2 that such a process involves no other bodies but the heat reservoirs means that once the transfer has taken place, these two are the only bodies experiencing a change in condition.

A heat bath or heat reservoir is, by definition, a constant temperature body capable of exchanging heat but not work. The common constraints associated to the no-work-exchange condition makes of a heat reservoir a body of constant volume -no *PV* work, of constant composition -no chemical work, and of constant and uniform temperature throughout its mass, qualities achieved by extremely large values of its thermal mass (mass times specific heat) and thermal conductivity so no internal temperature gradients, and consequently no work production from these gradients, are at all possible

The previous considerations allow us to realize that for such a body the expression for the first law reduces to the form $dU = dQ$ which evinces the fact that the heat dQ exchanged by a heat reservoir, on reason of being identical to the change of its internal energy -a function of state- is independent of the path, and in being this so the entropy change of this body is determined, apart of its temperature T, by the said actual amount of heat dQ by it exchanged.

In the words of Denbigh:

> The change of state of the heat bath is thus *entirely determined* by the heat transfer...and (its entropy change) is therefore the same whether this transfer is reversible or not. (1968, Footnote, p. 40).

In accord with the previous considerations, the entropy changes experienced by the hot (*hr*) and cold (*cr*) reservoirs in the irreversible heat transfer process of Figure 2 will be written as follows:

$$\Delta S_{hr} = -Q_h/T_h \qquad (22)$$

$$\Delta S_{cr} = Q_h/T_c \qquad (23)$$

Combined, equations (22) and (23) quantify the total-entropy change for the said process in the following manner:

$$\Delta S_{total}[Q_h(T_h) \rightarrow Q_h(T_c)]_{irr} = -(Q_h/T_h) + (Q_h/T_c) \qquad (24)$$

The entropy changes given by equations (22) and (23) for the hot and cold reservoirs correspond, it should be noted, to changes in condition of these bodies expressed, respectively, by the following internal energy changes: $\Delta U_{hr} = -Q_h$, and $\Delta U_{cr} = Q_h$.

7. From one cycle of a reversible heat engine to a couple of irreversible transformations of heat

In Figure 3, (implicit) reversible engine (a) is shown transferring to the cold reservoir the portion Q_c of the cyclical amount of heat Q_h received by it from the hot reservoir of temperature T_h, as well as transforming the remaining portion Q into an equivalent amount of work W. It is understood that $Q_h = Q_c + Q$, and $Q = W$. The concatenation of one cycle of this engine's operation with that process shown in 3(*b*) in which the work output (W) of the engine is irreversibly transformed via a frictional mechanism into the equivalent amount of heat Q at the temperature of the cold reservoir, reduces

the said concatenation to the two irreversible transformations of heat depicted in $3(c)$.

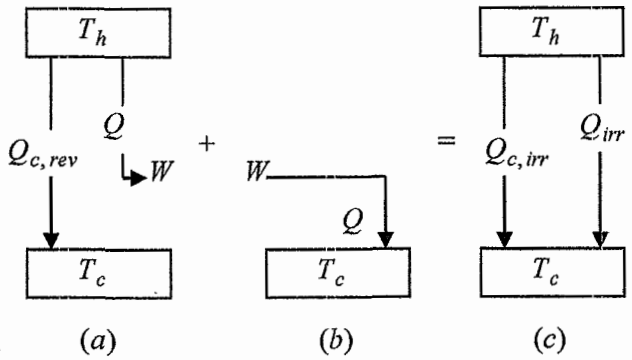


Figure 3. Out of the cyclical amount of heat Q_h taken in from the hot reservoir of temperature T_h, the reversible heat engine implicit in (a) transfers the portion Q_c to the cold reservoir of temperature T_c, and transforms the difference $Q = Q_h - Q_c$ into the equivalent amount of work W. The subsequent frictional degradation of W into heat at the temperature of the cold reservoir shown in (b) reduces the effects of concatenation $(a) + (b)$ to those produced by the two irreversible heat transfers shown in (c) in which $Q_{c,irr} = [Q_c(T_h) \rightarrow Q_c(T_c)]_{irr}$ and $Q_{irr} = [Q(T_h) \rightarrow W]_{rev} + [W \rightarrow Q(T_c)]_{irr} = [Q(T_h) \rightarrow Q(T_c)]_{irr}$

Using a notation similar to that used above for the transformations associated to a reversible engine, the transformations shown in $3(b)$ and $3(c)$ will be designated, from left to right, in the following manner: $[W \rightarrow Q(T_c)]_{irr}$, $[Q_c(T_h) \rightarrow Q_c(T_c)]_{irr}$, and $[Q(T_h) \rightarrow Q(T_c)]_{irr}$.

The said transit from $[3(a) + 3(b)]$ to $3(c)$ finds explanation in the following two considerations:

1. The final and sole effect of the concatenation of the heat-to-work transformation $[Q(T_h) \rightarrow W]_{rev}$ shown in $3(a)$, with the irreversible work-to-heat transformation $[W \rightarrow Q(T_c)]_{irr}$ shown in $3(b)$, is the irreversible transfer of Q from the hot to the cold reservoir: $[Q(T_h) \rightarrow Q(T_c)]_{irr}$. Having left the hot reservoir of temperature T_h to be transformed into W in $3(a)$, Q

reappears in $3(b)$ entering the cold reservoir following the frictional transformation of W back into heat there taking place. Therefore:

$$[Q(T_h) \rightarrow W]_{rev} + [W \rightarrow Q(T_c)]_{irr} = [Q(T_h) \rightarrow Q(T_c)]_{irr} \qquad (25)$$

2. Just like the availability of W in $3(a)$ is the reason behind the reversible condition of the two transformations there taking place —the feeding of W to the inverse cycle has the effect of restoring the initial condition without additional changes setting in (Íñiguez, 2020); It is the unavailability of work at the conclusion of work-degrading step $3(b)$ what explains the new irreversible condition of the two transformations of heat into which the combination $3(a) + 3(b)$ has been reduced to, namely $[Q_c(T_h) \rightarrow Q_c(T_c)]_{irr}$ and $[Q(T_h) \rightarrow Q(T_c)]_{irr}$. The reason is simple. The transfer back of Q_{irr} or $Q_{c,irr}$ or both from the cold to the hot reservoir demands the expenditure of work. The fact that none is now available means that the only possibility left for restoration of the initial condition without additional changes setting in, is represented by the spontaneous transit of these amounts of heat from the cold to the hot reservoir; the fact that this possibility is denied by experience explains their irreversible conditions.

The fact that $Q_h = Q_c + Q$ means that the changes of state which measured in terms of internal energy are experienced by the heat reservoirs participating in the processes depicted in Figure 3(c), namely $\Delta U_{hr} = -Q_h$, and $\Delta U_{cr} = Q_h$, are identical to those respectively experienced by the reservoirs involved in the process of Figure 2. Given the state function nature of the entropy, this identity in internal energy changes means, in turn, that the individual entropy changes as well as the combined total-entropy change for the reservoirs in 3(c) should also be given by equations (22), (23) and (24).

That no other change remains at the conclusion of concatenation $3(a) + 3(b)$ but the ones described above for the heat reservoirs, finds the following explanation in regard to the two other bodies taking part in it; 1) No change remains in the variable body on reason of the fact that at the end of each and every cycle it returns to its initial condition, and 2) No change remains in the work reservoir as the work originally deposited there by engine $3(a)$ has been retrieved from it and transformed into heat by process $3(b)$.

8. Proof of the negentropic nature of the transformation of heat into work

In terms of the irreversible heat exchanges taking place between the heat reservoirs, the total-entropy change for 3(c) can be expressed in the following manner:

$$\begin{aligned} &\Delta S_{total}[3(c)]_{irr} \\ &= \Delta S_{total}[Q_c(T_h) \rightarrow Q_c(T_c)]_{irr} \\ &+ \Delta S_{total}[Q(T_h) \rightarrow Q(T_c)]_{irr} \end{aligned} \qquad (26)$$

The previously noted identity between the total-entropy change of $3(c)$ with the one given by equation (24) for the process of Figure 2, allows us to write:

$$\begin{aligned} &\Delta S_{total}[3(c)]_{irr} \\ &= \Delta S_{total}[Q_c(T_h) \rightarrow Q_c(T_c)]_{irr} \\ &+ \Delta S_{total}[Q(T_h) \rightarrow Q(T_c)]_{irr} \\ &= Q_h(T_h - T_c)/(T_h T_c) \end{aligned} \qquad (27)$$

The fact that $Q_h = Q_c + Q$ permits the following re-expression of equation (27)

$$\begin{aligned} &\Delta S_{total}[3(c)]_{irr} \\ &= \Delta S_{total}[Q_c(T_h) \rightarrow Q_c(T_c)]_{irr} \\ &+ \Delta S_{total}[Q(T_h) \rightarrow Q(T_c)]_{irr} \\ &= (Q_c + Q)\left[\frac{T_h - T_c}{T_h T_c}\right] \end{aligned} \qquad (28)$$

The fact that apart from the temperatures, the total-entropy changes for the transfers of $Q_{c,irr}$ and Q_{irr} can only be functions of Q_c, and Q, respectively, leads to the following identifications:

$$\Delta S_{total}[Q(T_h) \rightarrow Q(T_c)]_{irr} = -(Q/T_h) + (Q/T_c) \qquad (29)$$

And

$$\Delta S_{total}[Q_c(T_h) \rightarrow Q_c(T_c)]_{irr} = -(Q_c/T_h) + (Q_c/T_c) \qquad (30)$$

It has been explained above that the only bodies affected by the concatenation of $3(a)$ and $3(b)$ are the heat reservoirs; this on reason of the return to their respective original conditions of all the other bodies involved in those processes, namely the working substance and the mechanical reservoir. This way, the changes experienced by the heat reservoirs in $3(c)$ become the embodiment of the totality of the effects of the said concatenation. The

irreversibility of the two transformations of heat to which the said concatenation reduces to, finds explanation in the unavailability of work at the end of 3(b); On the other hand, their total-entropy changes, as given by equations (29) and (30), find validation -via the state function nature of the entropy- from the identity of effects between the processes of Figures 3(c) and (2); with the total-entropy change of the latter safely anchored on the validity of the notion from which it was obtained, namely the form adopted by the first law of thermodynamics under the constraint $W = 0$ on which heat reservoirs operate. These first principles: the entropy state function (taken here as such), the definition of the heat reservoir, and the first law, are the only notions providing support to the present argument.

The total-entropy change for the irreversible transfer of Q depicted in 3(c) can, in attention to equation (25), be written as follows

$$\begin{aligned} &\Delta S_{total}[Q(T_h) \rightarrow Q(T_c)]_{irr} \\ &= \Delta S_{total}[Q(T_h) \rightarrow W]_{rev} \\ &+ \Delta S_{total}[W \rightarrow Q(T_c)]_{irr} \end{aligned} \qquad (31)$$

The further combination of the previous equation with equation (29) produces:

$$\begin{aligned} &\Delta S_{total}[Q(T_h) \rightarrow W]_{rev} + \Delta S_{total}[W \rightarrow Q(T_c)]_{irr} \\ &= -(Q/T_h) + (Q/T_c) \end{aligned} \qquad (32)$$

The evident facts that the entropy changes of the transformations represented from left to right in the left-hand side of the previous equation can only be functions of the amount and temperature of the heat transformed into work, or generated out of work i.e., $\Delta S_{total}[Q(T_h) \rightarrow W]_{rev} = f(Q, T_h)$ and $\Delta S_{total}[Q(T_c) \rightarrow W]_{irr} = F(Q, T_c)$ allows us, via a term-by-term comparison across the equal sign in this equation, to perform the following identifications:

$$\Delta S_{total,NT}[Q(T_h) \rightarrow W]_{rev} = -Q/T_h \quad \blacksquare \qquad (33)$$

and:

$$\Delta S_{total,NT}[W \rightarrow Q(T_c)]_{irr} = Q/T_c \qquad (34)$$

The '*NT*' (for Negentropic Thermodynamics) subindex added to the two previous equations identify their total-entropy changes as the ones coming out of our own correction of post-Clausius' thermodynamics.

9. The failure of the law of increasing entropy and the inevitability of the negentropic nature of $dS[dQ(T) \rightarrow dW]_{rev}$

In order to exhibit the incapability of the law of increasing entropy to explain the total-entropy change for $[Q(T_h) \rightarrow Q(T_c)]_{irr}$ as it takes place in Figure 3(c), it is required, in addition to the *fiat* $\Delta S_{total,PCT}[Q(T_h) \rightarrow W)]_{rev} = 0$, of that other notion which identical to our just proven value given by eq. (34), forms also part of Clausius', and post-Clausius thermodynamics:

In the words of Clausius:

> If...a quantity of heat Q is generated by any process such as friction, and this is finally imparted to a body of temperature T, the uncompensated transformation thus produced has the value Q/T. (1879, p. 214)

From Pitzer and Brewer:

> If an amount of work δW is degraded to heat at temperature T, the increase in entropy is $\delta W/T$ (1961, pp. 85-86)

An equivalent and even more precise statement than the previous one is given below:

> ...If we now allow the weight to sink back to its original level and if its energy is simultaneously transformed into heat by friction, the heat of friction $Q_f = W$ being added to the source T_0, the entropy of the latter will increase by $\Delta S = Q_f/T_0 = W/T_0$. (Schmidt, 1966, pp. 130-131)

In transformation notation, the previous statements find the following representation

$$\Delta S_{total,PCT}[W \rightarrow Q(T_c)]_{irr} = Q/T_c \qquad (35)$$

The failure of the law of increasing entropy comes to light when these two notions, namely $\Delta S_{total,PCT}[Q(T_h) \rightarrow W)]_{rev} = 0$ and $\Delta S_{total,PCT}[W \rightarrow Q(T_c)]_{irr} = Q/T_c$, are used to calculate, via equation (31), the total entropy change for the irreversible transformation of Q taking place in 3(c):

$$\begin{aligned} &\Delta S_{total,PCT}[Q(T_h) \rightarrow Q(T_c)]_{irr} \\ &= \Delta S_{total,PCT}[Q(T_h) \rightarrow W]_{rev} + \Delta S_{total,PCT}[W \rightarrow Q(T_c)]_{irr} \\ &= 0 + Q/T_c = Q/T_c \qquad (36) \end{aligned}$$

This way, instead of producing the correct total-entropy change for the irreversible transformation of Q, namely $-(Q/T_h)+(Q/T_c)$, the body of knowledge subsumed by the law of increasing entropy assigns to this process the blatantly incorrect total-entropy change of Q/T_c.

The formalism supporting the *LIE* requires the negation of the state function nature of the entropy, and with it the whole of the thermodynamics of the second law, as the very same process, the irreversible transformation of Q taking place in Figure 3(c), will be assigned two different total-entropy changes, one, given by eq. (32), in agreement with that known to correspond to an irreversible transformation of heat, namely $\Delta S_{total}[Q(T_h) \rightarrow Q(T_c)]_{irr} = -(Q/T_h)+(Q/T_c)$; the other one, given by eq. (36), $\Delta S_{total,PCT}[Q(T_h) \rightarrow Q(T_c)]_{irr} = Q/T_c$, in accord with the patently false notion that $\Delta S_{total,PCT}[Q(T_h) \rightarrow W]_{rev} = 0$.

The argument leading to equation (36) can be generalized by saying that the assignment to $\Delta S_{total}[Q(T_h) \rightarrow W]_{rev}$ of *any* value different from $-Q/T_h$ will lead to a similar conclusion i.e., to an irreversible heat transfer with an entropy change different from the one it is known to be associated with. This fact evinces the inevitability of equation (33)

10. $\Delta S_{total,NT}[Q(T_h) \rightarrow W]_{rev} = -Q/T_h$ and the correct total-entropy change for reversible heat engines.

If in the reversible cycle *ABCDA* of Figure 1(a) we assume a zero total-entropy change at the beginning (*i*) of the operation, i.e., $S_{total}[A_i] = 0$ then, at the conclusion of isothermal and reversible process *AB* we will have $S_{total}[B] = S_{total}[A_i] + \Delta S[AB] = -W_h/T_h$. Recall here that at the conclusion of this process the amount of heat Q_h of temperature T_h has been transformed into an equivalent amount of work W_h with an associated entropy change, in accord with eq. (33), of $-W_h/T_h$.

In adiabatic and reversible transit *BC*, even if there is no change in the entropy of the gas, there is however a modification -a correction, if you will- on the entropy change associated to the transformation of heat into work. The reason is actually very simple. When isentropic process *BC* disengages the work-producing system -the cylinder/piston device and the gas by it contained, as well as the mechanical reservoir- from the hot reservoir and engages it to the cold reservoir, the work attached to this now cold system becomes indistinguishable from an identical amount of work generated at T_c, and on reason of this in the transit *BC* the entropy effect associated to W_h changes

from $-W_h/T_h$ which is the one corresponding to T_h, for $-W_h/T_c$ which is the one that would have been produced had this amount of work been generated at T_c. Since no other effect but this is produced in this *BC* transit, we can then write $S_{total}[C] = -W_h/T_c$. The quality of heat, it should be remembered, changes with its temperature. (Smith and Van Ness, 1965, pp. 175-176)

Let us now retrieve W_c from the mechanical reservoir in order to drive with it isothermal and reversible compression *CD* which is to take to gas to condition *D*. The total entropy change for this process can, in accordance with eq. (34), be written as $\Delta S_{total}[CD] = W_c/T_c$, in which case the total-entropy at D amounts to $S_{total}[D] = S_{total}[C] + \Delta S_{total}[CD] = (-W_h/T_c) + (W_c/T_c) = -W/T_c$, where $W = W_h - W_c$ represents the work remaining at this point in the mechanical reservoir, and as such, it represents the work output of the cycle.

In opposition to the 'correction' effect previously described in regard to process *BC*, we find isentropic process *DA* disengaging from the cold reservoir what was previously referred to as the work-producing system, and engaging it to the hot reservoir. In this new condition, as part of the hot isotherm, W becomes indistinguishable from an identical amount of work generated at T_h and if so, its negentropy transits from $-W/T_c$ to $-W/T_h$. Since no other effect but this is introduced by *DA* we can write $S_{total}[A_f] = -W/T_h$, where *f*, for final, means the value of the total-entropy at point *A* at the completion of the cycle *ABCDA* or, equivalently, the total entropy change for one cycle in the operation of reversible engine 1(a). Therefore:

$$\Delta S_{total,NT}[\text{heat engine}]_{rev} = S_{total}[A_f] - S_{total}[A_i] = -W/T_h \qquad (37)$$

An identical result is obtained if we assume that the cyclical evolution of the ideal gas is to start at point *E* as shown in Figure 1(a). This point, it should be remembered, was chosen to assure identity between the amount of heat taken in by the gas from the hot reservoir along *EB*, with the one released by it to the cold reservoir along *CD*. Let us then define $S_{total}[E_i] = 0$ and also recognize that at the conclusion of process *EB* the total entropy at *B* will be equal to $S_{total}[B] = S_{total}[E_i] + \Delta S[EB] = -W_c/T_h$. Here W_c is the work produced out of an equivalent amount of heat Q_c of temperature T_h. At the conclusion of process *BC*, and for the same reason explained above, we will find the total entropy at *C* being given by $S_{total}[C] = -W_c/T_c$. Isothermal

and reversible compression *CD* takes now place with the consumption of the whole of the work W_c available in the mechanical reservoir. This work ends up in the cold reservoir in the form of its equivalent amount of heat Q_c. According to this, the total-entropy change at *D* will amount to: $S_{total}[D] = S_{total}[C] + \Delta S_{total}[CD] = -W_c/T_c + W_c/T_c = 0$. The fact that *D* finds the gas in contact with the cold reservoir and attached to a depleted mechanical reservoir means that along *DA* no temperature correction takes place and if so, we will have $S_{total}[A] = S_{total}[D] = 0$. If we take a closer look at the series of operations which starting at *E*, conclude in *A*, we will realize that they define the reversible transit of the amount of heat Q_c from the hot to the cold reservoir, and that according to our results, this transformation has an associated entropy change of:

$$\Delta S_{total,NT}[Q_c(T_h) \rightarrow Q_c(T_c)]_{rev} = 0 \qquad (38)$$

Let us now recognize that the process required to bring the cycle to its completion is reversible and isothermal expansion *AE*. In it the amount of heat Q, $Q = Q_h - Q_c$ is transformed into the equivalent amount of work W with an associated entropy change, in accord with eq. (33), of $-Q/T_h$. This result, written as $S_{total}[E_f] = -Q/T_h$ allows us to write the total-entropy change for one cycle in the operation of reversible engine 1(a) as

$$\begin{aligned}\Delta S_{total,NT}[\text{heat engine}]_{rev} &= S_{total}[E_f] - S_{total}[E_i] \\ &= -Q/T_h = -W/T_h \qquad (39)\end{aligned}$$

result that confirms the one given above in eq. (37) and that, contrary to common dogma, asserts that reversible engines are negentropic in an amount given by the quotient of their work output and the temperature of its hot reservoir:

$$\Delta S_{total,NT}[\text{heat engine}]_{rev} = -W/T_h \qquad (40)$$

If we add zero to this equation in the form of $[-(Q_h/T_h) + (Q_c/T_c)]_{rev}$ we will get the following equation:

$$\Delta S_{total,NT}[\text{heat engine}]_{rev} = -\frac{Q_h}{T_h} + \frac{Q_c}{T_c} - \frac{W}{T_h} \qquad (41)$$

Having written, in compliance with the mandate of the *LIE*, the total-entropy change for a reversible heat engine using only the first two terms on the right-hand side of the previous equation, the noted thermodynamicists Pitzer and Brewer explained the absence of the other term by saying: "The work term

does not appear in the... equation, since it involves no entropy..." (1961, p. 95)

11. Discussion

A rigorous thermodynamic proof has been presented to the effect that reversible transformation $[Q(T_h) \rightarrow W]_{rev}$ takes place with a total-entropy change of $-Q/T_h$. This result led us, in turn, to a zero total-entropy change for the other transformation associated by Clausius to the operation of reversible heat engines, namely the transformation of heat from the hotter to the colder reservoir, or vice versa.

Our results find the following expressions:

$$\Delta S_{total,NT}[Q(T_h) \rightarrow W]_{rev} = -Q/T_h \qquad (42)$$

$$\Delta S_{total,NT}[W \rightarrow Q(T_h)]_{rev} = Q/T_h \qquad (43)$$

$$\Delta S_{total,NT}[Q_c(T_h) \rightarrow Q_c(T_c)]_{rev} = 0 \qquad (44)$$

$$\Delta S_{total,NT}[Q_c(T_c) \rightarrow Q_c(T_h)]_{rev} = 0 \qquad (45)$$

The comparison between equations (2) and (42) shows that the total-entropy-change assigned by *NT* to transformation $[Q(T_h) \rightarrow W]_{rev}$ agree with that of Clausius. The comparison between equations (10) and (44) shows, on the other hand, the agreement between our value for $[Q(T_h) \rightarrow Q(T_c)]_{rev}$ and that of post-Clausius thermodynamics. These agreements, as should be obvious, also apply for the respective inverses of these transformations.

None of the two formulations, Clausius' thermodynamics, and post-Clausius' thermodynamics came out with the correct values for the whole of the transformations involved in reversible engines and refrigerators.

No longer tasked with providing support to a law which having originated in a flawed argument, became later on 'the law' by the force of authority, the thermodynamics of the second law takes its proper place as an accounting device for the net-quality change of the energy transformations taking place in thermodynamic processes.

The certainties provided by post-Clausius' thermodynamics are no more. Nature is much more complex than the idiosyncrasies and prejudices of some would like it to be. The consequences, scientific and otherwise, coming out of our results might be unsettling for many.

Just like the way Heraclitus understood the becoming of this universe of ours, thermodynamic processes are now revealing to us what has been their perpetual *modus operandi*, the battlefield of the dialectical opposites, entropy and negentropy.

Leaving behind our needed beliefs and certainties is, and rightfully so, not an easy job:

> I know that most men –not only those considered clever, but even those who are very clever, and capable of understanding most difficult scientific, mathematical, and philosophical problems– can seldom discern even the simplest and most obvious truth if it be such as to oblige them to admit the falsity of conclusions they have formed, perhaps with much difficulty –conclusions of which they are proud, which they have taught to others, and on which they have built their lives. (Tolstoy, 1899, p. 124)

In the case of new scientific notions these must be taken to what Planck calls

> ...the highest court of appeal –experience. Whatever the decision may be, lasting gain will accrue to us from such a proceeding, since thereby we serve the chief end of natural science –the enlargement of our stock of knowledge." (1990, p. 107)

It is the opinion of this author that in negating the negentropic effect of the transformation of heat into work, the thermodynamics of the second law threw away the only analytical tool capable of making sense of self-organizing phenomena.

It is in fact in this field, with the self-organizing phenomenon known as Rayleigh-Bénard convection, where we will seek the judgement of experience to the results of our effort. A Thermodynamic model for this phenomenon has been constructed based precisely in the opposite effects of energy-degrading and energy-upgrading. In the second and last paper of this series we will describe its development and the results of its first contrasts with experimental data.

12. Final comment

Everything this author knows about the transition from Clausius' to post-Clausius' thermodynamics (Íñiguez 2021b) indicate that the goal of the correctors was to change whatever was changeable in Clausius' work, as long as its conclusion, namely the law of increasing entropy, remained valid. The

impasse represented by the lack of a proof for the zero total-entropy change of the transformation of heat into work was no deterrent for them: they simply used the sword of their authority to cut through a *Gordian knot* they were in any other way, say using science, reason and logic, incapable of untying: they simply declared $dS_{total,PCT}[dQ(T) \rightarrow dW]_{rev} = 0$ to be true. With this *fiat* in place, the total-entropy change for an isothermal and reversible process, reduced by post-Clausius' thermodynamics to a simple reversible heat transfer process between two bodies of identical temperatures, became equal to zero (Klotz and Rosenberg, p. 133; Bent, pp. 78-79), and with this new notion, the total-entropy changes of the two transformations taking place in reversible engines, transited from the non-zero values originally assigned to them by Clausius, to the zero values prescribed by the law of increasing entropy.

The following quotes provide elements of judgement that might help us answering the question: 'why transform the thermodynamics of the second law into a parody of a scientific construction just to keep the validity of the law increasing entropy intact?' The following two quotes evince what was at play when all this was happening:

> The (second) law has caught the attention of poets and philosophers and has been called the greatest achievement of the nineteenth century. Engels disliked it, for it supported opposition to Dialectical Materialism, while Pope Pius XII regarded it as proving the existence of a higher being (I.P. Bazarov, Wikiquote).

Finn is even more precise in his appreciation about this issue:

> Traditional Christian concepts appear to have deeply influenced many of the scientists who stated and developed the second law as a universal principle. To cite Lord Kelvin (William Thomson), one of the originators of the second law, the law meant that "...all motion except that of heat must have an end, unless it pleases God to restore by an act of new creative power the dissipation of mechanical energy that always goes on...Seen in its historical context, the law seems to be less a law and more of a daring...designed to be a contender in the running cosmological debates of its time (Finn, pp. viii, 71).

A detailed discussion on this issue can be found in (Íñiguez 2021b)

References

Angrist, S. W. and Hepler, L. G. 1967, *Order and Chaos*, Basic Books, NY.

Barrow, G. M. 1973. *Physical Chemistry*. McGraw-Hill, NY.

Bazarov, I. P. 1964. *Thermodynamics*. Macmillan, NY. pp. 75-78 https://en.wikiquote.org/wiki/Thermodynamics

Bent, H. A. 1965. *The Second Law*. Oxford, NY.

Clausius, R. 1862, On the Application of the Theorem of the Equivalence of Transformations to the Internal Work of a mass of Matter, *Phil. Mag.* S. 4. Vol. 24. No. 159/160 (August/September 1862), pp. 81-97 and 201-213

Clausius, R. 1879. *The Mechanical Theory of Heat*. Macmillan, UK.

Denbigh, K. 1968. *The Principles of Chemical Equilibrium*. Cambridge, UK.

Dickerson, R. E. 1969. *Molecular Thermodynamics*. W. A. Benjamin, NY.

Eddington, A. S. 1929. *The Nature of the Physical World*. The Macmillan Co. UK.

Finn, C. B. 1974. Religion, Philosophy and the Second Law of Thermodynamics, M. A. Dissertation, Department of Sociology, College of William and Mary, Williamsburg, VA.

Haxton, B. 2003. *Heraclitus*, Penguin Classics, NY.

Íñiguez, J. C. 2020. An irreversible heat engine working at the reversible efficiency. *International Journal of Chemistry*. Vol. 4, No. 2, doi: 10.5539/ijc.v12n2p1

Íñiguez, J. C. 2021a. The egregious error in Clausius' argument leading to the law of increasing entropy, https://www.researchgate.net/profiles/Jose-Iniguez-3

Íñiguez, J. C. 2021b. On Clausius', post-Clausius', and Negentropic Thermodynamics, *Journal of High Energy Physics, Gravitation and Cosmology*, Vol. 7, No. 4. An extended version is available at https://www.researchgate.net/profiles/Jose-Iniguez-3

Klotz, I. M. and Rosenberg, I. M. 1986. *Chemical Thermodynamics*. Benjamin/Cummings, CA.

McGlashan, M. L. 1966. The Use and Misuse of the Laws of Thermodynamics, *J. Chem. Educ.*, Vol. 43, No. 5, pp. 226-232

Moore, W. J. 1972. *Physical Chemistry*. Prentice-Hall, NJ.

Norton, J. D. 2016. The impossible process: thermodynamic reversibility. *Studies in History and Philosophy of Modern Physics*, 55, pp. 43-61: doi:10.1016/j.shpsb.2016.08.001

Pitzer, K. S. and Brewer, L. 1961. *Thermodynamics,* McGraw-Hill, NY.

Planck, M. 1990. *Treatise on Thermodynamics*. Dover, NY.

Schmidt, E. 1966. *Thermodynamics*. Dover, NY.

Smith, J.M. & Van Ness, H. C. 1965. *Introduction to Chemical Engineering Thermodynamics,* McGraw-Hill, NY.

Tolstoy, L. 1899. *What is Art*? Thomas Y. Crowell & Co., NY.

Weber, H. C. and Meissner, H. P. 1957. *Thermodynamics for Chemical Engineers*, John Wiley & Sons, Inc., NY.

HADRONIC JOURNAL 45, 297 - 332 (2022)
DOI 10.29083/HJ.45.03.2022/SC297

NEGENTROPIC THERMODYNAMICS II: AN AUTOPOIETIC, HEAT-ENGINE-CENTERED MODEL FOR RAYLEIGH-BÉNARD CONVECTION

José C. Íñiguez
Retired Instructor of Science and Mathematics
1227 21st Street
Douglas, AZ 85607
iniguez.jose@gmail.com

Received May 28, 2022

Abstract

A thermodynamic model for the emergence and permanence of the convective structure in Rayleigh-Bénard convection (*RBC*) is here presented having an autopoietic concatenation of optimally-efficient heat engines as its central component. The model makes use of the proof advanced in Part I of this two-part series about the negative total-entropy change associated to the energy-upgrading process represented by the transformation of heat into work. This notion, under the assumption of the convective structure being the manifestation of the work output of the said concatenation of optimally efficient engines, allows for the development of an expression for its negative entropy change. This way the total-entropy equation for *RBC* ends up being composed of two oppositely-signed entropy terms: a positive one originating in the irreversible heat flow, and a negative one from the architectural and functional order of the convective structure. Along the path of increasing temperature gradients to which the fluid in *RBC* is subjected to, a temperature gradient is reached at which the fluid organizes itself in the form of cells or parallel rolls. This chaos-order transition is identified by our model as the point at which the entropic contribution to the total-entropy change is precisely balanced or neutralized by negentropy of the structure, producing as a consequence the transition from an entropic to a nonentropic universe. The nonentropic universe remains as long as structure-mediated-convection remains the only heat transfer mechanism. The span of life of the convective structure is this way represented by the interval of gradients defining the nonentropic path. These notions are put to the test by means of the experimental data of Schmidt and Saunders, and Schmidt and Milverton. The results obtained agree with our conceptualization

Keywords: Rayleigh-Bénard convection, autopoietic model, heat engines, entropy, negentropy, nonentropic interval of gradients, model predictions, contrast with experiment.

Copyright © 2022 by Hadronic Press, Inc., Palm Harbor, FL 34684, USA

1. Antecedents

From the Non-linear Physics Group of the University of Toronto we learn that:

> The simplest and most studied version (of Self-organization) is Rayleigh-Bénard convection...Here the fluid...is confined between rigid plates (no free surface) which are held at constant temperature, with the hot plate below. In 1916...Lord Rayleigh showed that an initially motionless fluid layer becomes unstable to small flow perturbations when the temperature difference is sufficiently large...Flow perturbations grow when the buoyancy forces on a perturbed parcel of fluid become sufficient to overcome dissipation due to viscosity and thermal diffusion...The flow pattern consists of rolls or hexagonal cells, depending on the details of the temperature dependence of the fluid properties.
> (https://www.physics.utoronto.ca/nonlinear/)

In this regard Feistel and Ebeling state that:

> Bénard cell transforms above a critical temperature gradient thermal energy into the mechanical energy of hydrodynamic flows etc. In this way our idea is, that dissipative structures may be considered from a thermodynamic point of view as energy transforming machines...The theoretical restriction to the coefficient of efficiency of this energy change is given by the second law...(the) structure is a kind of energy transforming engine which uses part of the energy inflow to produce a new form of energy which is of higher thermodynamic value (what means lower entropy). (1989, pp.82, 85)

A similar statement is that of Soltzberg:

> The most frequently referred to archetype of self-organization seems to be Bénard convection...when a shallow pool of liquid is heated from below, the onset of convection at a critical temperature difference between the lower and upper surface is accompanied by the spontaneous appearance of either a convection honeycomb pattern or parallel convection rolls. Although the organized state has a lower entropy than its homogeneous antecedent, the throughput of energy that maintains the organization produces an overall increase in the entropy of the universe, so the second law remains inviolate. (1989, p.187)

Being the optimally-efficient heat engine the prototype of energy-upgrading machines it came naturally, at the light of the previous considerations, to base our model for Rayleigh-Bénard convection (*RBC*) on these devices.

2. The hydrodynamic and thermodynamic approaches to *RBC*

The experiments which stimulated the study of thermal instability in horizontal layers of fluids heated from below were those of Bénard in 1900:

> In all cases, Bénard found that when the temperature of the lower surface was gradually increased, at a certain instant, the layer became reticulated and revealed its dissection into cells (Chandrasekhar, 1961, p. 60)

The principal facts established by these experiments were:

> ...*first*, a certain critical adverse temperature gradient must be exceeded before instability can set in; and *second*, the motion which ensue on surpassing the critical temperature gradient have a stationary cellular character...The theoretical foundations for a correct interpretation of the foregoing facts were laid by Lord Rayleigh. Rayleigh showed that what decides the stability, or otherwise, of a layer of fluid heated from below is the numerical value of the non-dimensional parameter $R = (g\alpha\beta/\kappa\nu)d^4$ where g denotes the acceleration due to gravity, d the depth of the layer, $\beta(= |dT/dz|)$ the uniform adverse temperature gradient which is maintained, and α, κ, and ν are the coefficients of volume expansion, thermometric conductivity, and kinematic viscosity, respectively; R as defined here is called the *Rayleigh number*. Rayleigh further showed that instability must set in when R exceeds a certain critical value R_c; and that when R just exceeds R_c, a stationary pattern of motions must come to prevail. (Chandrasekhar, 1961, p. 10)

The temperature gradient associated to R_c is the critical temperature gradient.

The field of study based on the works of Bénard and Rayleigh constitutes the hydrodynamic approach to the self-organization of fluids heated from below. Apart from the original works of Bénard (1900, 1901) and Rayleigh (1916), the book of Chandrasekhar (1961) constitutes an essential reference to this field. Up to date information might be found at:
https://www.physics.utoronto.ca/nonlinear/index.html

The discussions to be here presented, thermodynamic in nature, are based on the proof provided in Part I of this two-part series about the negentropic nature of the energy-upgrading process represented by the transformation of heat into work. The coupling of this notion with the one associating an entropic effect to the irreversible transformation of work into heat, as well as to the irreversible transfer of heat from a hotter to a colder body, constitutes the backbone of our model and of the equation obtained from it for the total-entropy change of Rayleigh-Bénard convection. At difference with the hydrodynamic approach, our criterions for emergence and demise of the convective structure are independent of the nature of the fluid and solely dependent on the temperatures defining the gradients associated to those two events. Figure 1 may be taken to represent the condition of the universe of *RBC* at emergence or at any other point along the path of permanence of the convective structure.

3. The defining characteristics of RBC's model

Figure 1 shows the coupling of engines with which the two changes taking place in Rayleigh-Bénard convection, namely the irreversible heat flow from T_h to T_c, as well as the maintenance of the convective structure, are here to be modeled. The essential characteristics of this coupling of optimally efficient engines are the following: 1) The temperatures of the outer, or limit heat reservoirs are T_h and T_c, with $T_h > T_c$. The difference between these temperatures defines either the critical temperature gradient associated to the emergence of the structure, or any of the gradients associated to the path in which structure-mediated convection remains the sole heat transfer mechanism; 2) An arbitrarily large number of optimally-efficient engines 1, 2, 3,...*n* and their associated heat reservoirs have been fitted between the outer heat reservoirs; 3) These engines have been arranged in a way such that the cold reservoir of any one of them acts also as the hot reservoir of the succeeding engine in the direction of decreasing temperatures; 4) The temperatures of the reservoirs are ordered in the following fashion $T_h > T_1 > T_2 \ldots > T_c$. These temperatures have been chosen in such a way that the efficiency of any one of these optimally efficient engines is identical to that of any other; 5) The cyclical amount of heat Q_h delivered by the source (the reservoir of temperature T_h) reaches the heat sink (the reservoir of temperature T_c) after passing through each and every one of the coupled engines i.e., no other path exists for heat to flow from the source to the sink; 6) In its cascading transit along the coupling this heat is made to provide a

combined amount of work nw, where n is the number of coupled engines. The identity in heat inputs and efficiencies explains the identity of the cyclical work outputs of the coupled engines; 7) The model identifies the structure's maintenance (renovating) mechanism as a work recycling step consisting in that while the work produced by any one engine incorporates to the convective structure, becoming this way part of its architectural and functional order, an identical amount of work disincorporates (separates) from the structure and is fed as heat, alongside the heat passing through ($Q_h - w$), to its succeeding engine in a way such that any engine receives the same amount of heat Q_h as any other. The details of this mechanism are the matter of the discussion associated to Figure 2; 8) The minimum number (n) of engines required to produce an autopoietic coupling is $n = 2$.

As defined, the coupling agrees with the essential lines of Jantsch's autopoietic systems i.e., those that renew themselves in the same process-structure (1980, p.10).

As discussed elsewhere (Íñiguez, 2020), each of the engines of the coupling shown in Figure 1 can be described as an irreversible heat engine working at its maximum efficiency. The notion at the center of the previous consideration recognizes that optimal efficiency is a necessary but not a sufficient condition to guarantee reversibility in a heat engine. Reversibility, it should be remembered, refers to the possibility of restoring the exact initial condition once a change has taken place. As long as the work output of an optimally efficient engine remains available in a mechanical reservoir all we have to do to restore the exact initial condition is to retrieve this work and use it to propel the inverse cycle or, equivalently, to run the engine in reverse i.e., as a refrigerator. If, however, for whatever reason the work output of the engine is not available, restoration of the exact initial condition is impossible. In this case running the engine in reverse will not bring restoration on reason of the changes left behind in that body called on to supply the missing engine's work. In heat engines, as in any other work-producing process, reversibility demands both, the production of the maximum amount work compatible with its conditions of operation, as well as the work's permanence –its continued availability- in some mechanical reservoir. The fact that in the coupling of Figure 1 the engines' work output, in becoming the structure's architectural and functional order, is never available for restoration purposes, means that these optimally-efficient engines are irreversible at any moment of their operation, and if so, also irreversible is their coupling.

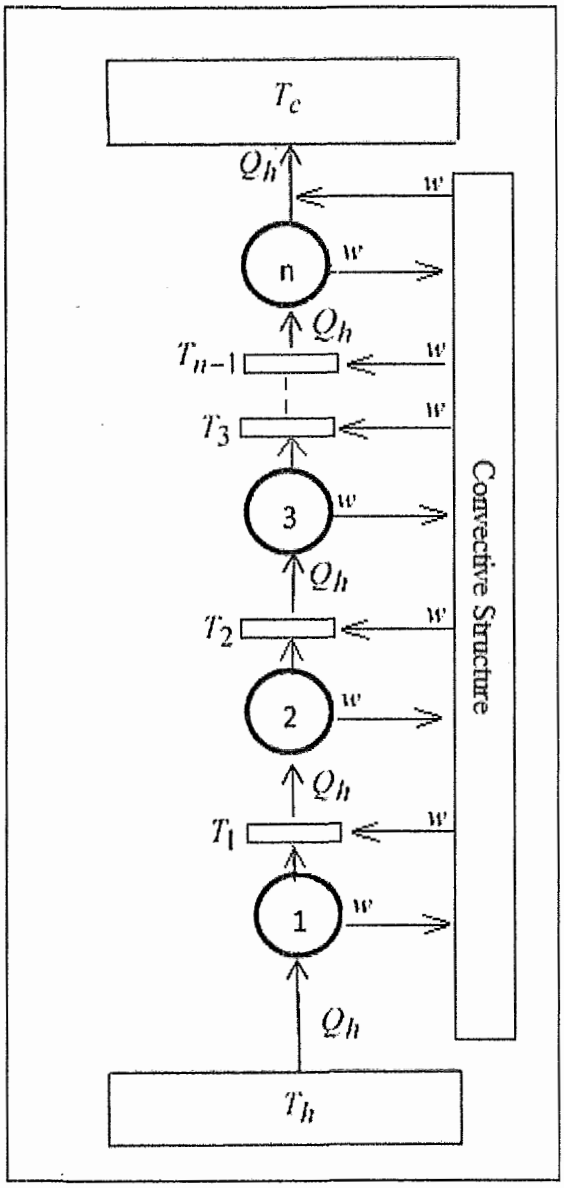


Figure 1. The autopoietic coupling of optimally efficient heat engines used as model for *RBC* convection. The heat passing through any of the engines, namely $Q_h - w$, is designated in the text as Q_c. Details of the work-recycling step are the matter of the discussion presented in reference to Figure 2.

If we are to write an equation for the total-entropy change of Rayleigh-Bénard convection at the critical point (or at any point along its path of permanence) we have to include the contributions of all the bodies in any way involved in this process. Thus understood the total-entropy change for *RBC* might also be designated the entropy change for *RBC's* universe.

Let us then start this task by recognizing that, as previously noted, "...the...structure is a kind of energy transforming engine which uses part of the energy inflow to produce a new form of energy which is of higher thermodynamic value (what means lower entropy)..." (Feistel and Ebeling, 1989, p. 85), or, equivalently, that "Although the organized state has a lower entropy than its homogeneous antecedent, the throughput of energy that maintains the organization produces an overall increase in the entropy of the universe, so the second law remains inviolate." (Soltzberg 1989, p. 187)

The previous authors, it should be noted, recognize two differently signed entropy effects in the universe of *RBC*: the entropic throughput of heat from the hot to the cold reservoir, and a negentropic contribution associated to the architectural and functional order of the structure. If so, the equation for the total-entropy change (ΔS_u) of *RBC* should take the following form: $\Delta S_u[RBC] = \Delta S[\text{heat transfer}] - \Delta S[\text{structure}]$.

In his quote Soltzberg asserts a larger than zero result for the previous equation, this on reason of the entropic contribution of the heat transfer overcoming the negentropic contribution of the organized state "...so the second law remains inviolate". That no reference is given in Soltzberg's statement to any theoretical or experimental work which could provide support to his assertion leaves us with no other choice but to trace its sustenance to what might be called faith on the increasing entropy dogma of the second law. As far as the present author's knowledge goes, there is no evidence of a determination, a calculation, or an estimation of the $\Delta S[\text{structure}]$ term ever having been made, and in being this so, no definitive statement can at all be advanced about the total entropy change for *RBC* or any other self-organizing phenomenon whatsoever. This issue won't be settled until a sound-science-based and experiment-backed result for $\Delta S[\text{structure}]$ has been produced. It is here where some of the essential tenets of the second law might find their definitive or crucial challenges.

In order to be able to produce a total entropy equation for the model of *RBC* displayed in Figure 1 we need to quantify some of its essential characteristics as they were described in Section 2.

Sections 4, 5, and 6, below are excerpts from a previously published work dealing with the efficiency of the coupling of Figure 1. (Íñiguez, 2010)

4. The common efficiency of the optimally efficient engines in RBC's model

As previously stated, the efficiency of any engine is equal to that of any other. If this common efficiency is represented as η, then

$$\eta_1 = \eta_2 = \eta_3 = \cdots = \eta_n = \eta \qquad (1)$$

In the previous equation $\eta_1, \eta_2, \eta_3 \ldots, \eta_n$ respectively represent the efficiencies of engines 1, 2, 3..., n of the coupling depicted in Figure 1.

Let us now recognize that in terms of the temperatures of the reservoirs, equation (1) leads to the following series of equations:

$$(T_h - T_1)/T_h = (T_1 - T_2)/T_1 = (T_2 - T_3)/T_2 = \cdots = (T_{n-1} - T_c)/T_{n-1} = \eta \qquad (2)$$

If starting with the leftmost term, one by one of the terms of the previous equation is made equal to η and respectively solved for $T_1, T_2, T_3 \ldots, T_c$ the following results are obtained:

$$T_1 = T_h(1-\eta), \quad T_2 = T_1(1-\eta), \quad T_3 = T_2(1-\eta), \ldots T_c = T_{n-1}(1-\eta) \qquad (3)$$

Substitution of the first of these equations in the second followed by substitution of the thus modified second in the third, and so on leads, in turn, to the following results:

$$T_1 = T_h(1-\eta),\ T_2 = T_h(1-\eta)^2,\ T_3 = T_h(1-\eta)^3 \ldots T_c = T_h(1-\eta)^n \qquad (4)$$

The reversible efficiency (η^*) prescribed by the second law for a single reversible engine working between T_h and T_c, can now be introduced through the following equation:

$$\eta^* = (T_h - T_c)/T_h \qquad (5)$$

Taking now the last member of the string of equations represented in (4), and equating it with the one produced when equation (5) is solved for T_c, produces the following result:

$$T_h(1-\eta)^n = T_h(1-\eta^*) \qquad (6)$$

Solving this last equation for η leads to

$$\eta = 1 - \sqrt[n]{1-\eta^*} = 1 - \sqrt[n]{T_c/T_h} \qquad (7)$$

The previous equation, it should be pointed out, subsumes the information required to actually set up these couplings. When given values for T_h, T_c, as well as a selected value for n are substituted in this equation, the common efficiency becomes known. This piece of information alongside T_h fixes, through the series of equations shown in (4), the temperatures of the intermediate reservoirs. Conversely, if in addition to T_h and T_c it is the common efficiency the one that is provided, then the number of engines required can also be determined through (7). As before, η alongside T_h will determine, through the series of equations given in (4), the temperature of the intermediate reservoirs.

Let us now transit from individual engine efficiencies to that of the coupling as a whole.

5. The coupling's efficiency

The efficiency of the coupling of optimally efficient engines of Figure 1, which from now on will be represented as η_{cc}, will be written as follows

$$\eta_{cc} = W/Q_h \qquad (8)$$

As Figure 1 makes evident, each of the coupled engines extract an identical amount of work w from their respectively identical heat input Q_h. It is the combined or total amount of work extracted along the coupling from this amount of heat what in equation (8) is represented by W. If so, then

$$W = n\,w \qquad (9)$$

The fact that the work output of a heat engine is determined by the product of its efficiency times the incoming heat ($w = \eta Q_h$) allows us to write equation (9) in the following form

$$W = n\,\eta\,Q_h \qquad (10)$$

The appropriate combination of equations (8) and (10) leads to

$$\eta_{cc} = n\,\eta \qquad (11)$$

The substitution of equation (7) in (11) permits writing the following equation for the efficiency of the coupling

$$\eta_{cc} = n\left(1 - \sqrt[n]{1-\eta^*}\right) = n\left(1 - \sqrt[n]{T_c/T_h}\right) \qquad (12)$$

It is worth noticing that upon substitution of $n = 1$ this equation, as expected, reduces to $\eta_{cc} = \eta^*$

6. Further considerations on the efficiency of the coupling

Let us consider the behavior of η_{cc}, as given by equation (12), at η^* limit values, that is, as $\eta^* \to 0$ and as $\eta^* \to 1$. For the former situation we will have that

$$\lim \eta_{cc_{\eta^*\to 0}} = n(1 - \sqrt[n]{1-0}) = n\left(1 - \sqrt[n]{1}\right) = 0 \qquad (13)$$

The previous result is the reflection of the fact that no engine operation is possible in the absence of a gradient. The efficiency of the coupling, alongside the efficiency of its constituent engines, will collapse to zero in the limit where $T_h = T_c$.

For the latter of the two situations noted above we will have that:

$$\lim \eta_{cc_{\eta^*\to 1}} = n(1 - \sqrt[n]{1-1}) = n\left(1 - \sqrt[n]{0}\right) = n(1-0) = n \qquad (14)$$

Even if the previous situation corresponds, it should be noted, to the physically unrealizable conditions of $T_c = 0$, or $T_h = \infty$, it nevertheless allows us a glimpse at the behavior of η_{cc} in this extreme.

The behavior of η_{cc} between these limits will be ascertained by taking the partial derivative of η_{cc} with respect to η^* at constant n. By doing this we get

$$(\partial\eta_{cc}/\partial\eta^*)_n = 1/(1-\eta^*)^{1-1/n} \qquad (15)$$

With regard to the right-hand side of equation (15), it should be noted that on reason of $0 < \eta^* < 1$ it follows that $0 < 1 - \eta^* < 1$; and that on reason of $2 \leq n < \infty$ it follows that $0 < 1 - 1/n < 1$. Being this so, then

$$0 < (1-\eta^*)^{1-1/n} < 1 \qquad (16)$$

Consequently

$$1/(1-\eta^*)^{1-1/n} > 1 \qquad (17)$$

Therefore

$$(\partial\eta_{cc}/\partial\eta^*)_n > 1, \;\; n \geq 2 \qquad (18)$$

The fact that the partial derivative came out positive indicates that η_{cc} is an increasing function of η^*. This means that as η^* increases its value in its

interval of definition, so does η_{cc}. On the other hand, the fact that the partial derivative is larger than one indicates that for every n, $n \geq 2$, it is true that $\eta_{cc} > \eta^*$ and the coupling's efficiency, defined by equation (8), has a larger value than that of a reversible engine working between T_h and T_c.

The previous considerations had to do with the effect that changes in η^* produce in η_{cc}. In what follows we will inquire at the effect that n has on η_{cc} at a given value of η^* or, equivalently, for given values for T_h and T_c. Let us then consider the following limiting behavior of equation (12)

$$\lim \eta_{cc_{n\to\infty}} = lim_{n\to\infty}\, n(1 - \sqrt[n]{1-\eta^*}\,) = -\ln(1-\eta^*) \qquad (19)$$

The previous result was obtained via L'Hôpital's rule (Anton, 1999, pp. 277-284) treating the discrete variable n as a continuous variable. In non-technical terms the rationale for this is that "...if you know the limit of a continuous variable...you also know the limit of any sequence you get by 'picking out' points of this function's domain...." (math.stackexchange.com). The use of equation (12) with $T_h = 300\text{K}$, $T_c = 297\text{K}$, temperatures comparable to those of Table 1, and a small number of n's: 10, 10^2, 10^3, 10^4, 10^5, and 10^6 served to confirm, via the following respective results, 0.01004528708, 0.1004983082, 0.01005028534, 0.0100503308, 0.01005033535, 0.01005033578 the limit value for η_{cc} obtained from equation (19) of magnitude 0.01005033585. The important thing to notice from equation (19) is the finite magnitude acquired by η_{cc} for an arbitrarily large number of coupled engines and, consequently, the finite magnitude of W for the same condition. A superficial look at equation (10): $W = n\,\eta\,Q_h$, might lead to the wrong conclusion that W is bound to be arbitrarily large for a correspondingly large number of coupled engines. That this is not so can be understood once we realize that as n grows arbitrarily large the inner gradients $T_h - T_1$, $T_1 - T_2$, $T_2 - T_3 \ldots T_{n-1} - T_c$, and consequently the common efficiency η, grows arbitrarily small, fact evinced by the following limit that makes use of the definition of η given by equation (7): $\lim_{n\to\infty}\eta = \lim_{n\to\infty}(1 - \sqrt[n]{1-\eta^*}) = 1 - (1-\eta^*)^{1/\infty} = 1 - 1 = 0$. Any increase in n is compensated by a corresponding decrease in η in a way such that η_{cc}, defined by equation (11) as $\eta_{cc} = n\eta$, remains finite for all n, and if so, also finite remains the coupling's work output given by $W = \eta_{cc}Q_h$. The previously annotated limit value of η_{cc} of magnitude $\eta_{cc} = 0.01005\ldots$ should be compared with the corresponding efficiency of a single reversible engine working between the same temperatures: $\eta^* = (300 - 297)/300 = 0.01000$.

Interpretation of the results previously obtained for η_{cc} such as the one given by equation (18) showing η_{cc} to be, for any $n \geq 2$, larger than the reversible efficiency of a single engine working between T_h and T_c, or the one just given comparing the limit value of η_{cc} with η^*, in apparent contradiction to Carnot's original version of the second law stating that no engine can be more efficient than a reversible engine working between the same temperatures (1824/1960, pp. 11-12, 19-20), requires us to keep in mind that η_{cc} represents the efficiency of the autopoietic coupling, and that this coupling is not a heat engine per se, this in the sense that it was designed not with the purpose of supplying mechanical energy for uses outside the system comparable to the lifting of weights, but with the intent of recycling it as heat and feeding it to the engine next in sequence once it structure-supporting role comes to an end. The fact that the coupling's supra-Carnot's work output is never available to be used in the same way the mechanical energy output of a regular heat engine is used, is what negates the possibility of using W to propel a reversible Carnot refrigerator in order to produce the transfer of heat from T_c to T_h as the sole result. Any manipulation, alteration or disruption of *RBC* with the futile intent of harnessing W for purposes different than those it is tasked with in *RBC*, will inevitably lead to the destruction of the structure, to the inevitable transformation of W into heat, and to the concomitant destruction of the autopoietic coupling; we will get all this, but no W to feed the said Carnot refrigerator. Even diverting the work output of a single engine from its intended purpose, if a way can at all be found to do this, will disrupt the coupling's concerted mechanism and destroy the structure.

The impossibility of using W to feed a Carnot's refrigerator is what makes its definition given in equation (8), and its comparison with η^* given in equation (18), compliant with the fundamental notion of the second law understood as "Heat cannot, of itself, pass from a colder to a hotter body." (Clausius, 1879, p. 78)

7. On the negentropic nature of the transformation of heat into work, and on the central notion of our model

1) Even if classical thermodynamics asserts, through the law of increasing entropy, the entropy producing quality of energy degrading processes such as the irreversible transformation of work into heat or the irreversible transfer of heat from a higher to a lower temperature, it paradoxically negates any effect associated to energy upgrading processes, whose foremost representative example is the transformation of heat into work. Our position

on this matter starts recognizing that heat is lower-quality energy than work. You'll agree with this assertion as soon as you compare the number of tasks you can do with electricity with those you can do with fire. The fact that an energy-upgrading process has an effect directly opposite to those in which energy is degraded points out to the notion that if the latter are to be characterized by positive entropy changes, so then the former must be characterized by negative entropy changes. In simpler words, if quality loss is entropic, quality gain is necessarily negentropic. After all, as Heraclitus has said "The way up is the way back" (Haxton, 2003, p. 45). From reasons that have remained hidden for a long time, and that appear to start coming to light in recent times, the superior quality of work understood in terms of its chaos-decreasing (chaos-ordering), and change-inducing capabilities when compared with those of the heat from which it originates, has been erased from any consideration in thermodynamic entropy balances. Current thermodynamics position on this matter, eloquently expressed by the dictum "*Quod non est in formula non est in mundo*" (von Bertalanffy, 1975, p. 7014) is to blame for what can be called an extended lack of knowledge about this issue, situation no doubt related to the absence of any discussions on this matter from all or almost all of thermodynamic textbooks and classrooms.

Our previously described intuitive-position was finally confirmed by our just developed proof about the negative total-entropy change associated to the transformation of heat into work (Part I of this two-part series). Based on the identity of effects existing between an irreversible heat transfer and the concatenation of an optimally efficient heat engine with a work-degrading step in which the work output of the engine is frictionally degraded into heat at the temperature of its cold reservoir, we were able to prove that entropy changes of $-Q/T$, and Q/T are respectively produced by the transformations of a certain amount of heat Q of temperature T into an equivalent amount of work W, and vice-versa.

It might be surprising to more than one reader to know that the results coming out of our just mentioned proof agree with Clausius' own position on this matter, who in reference to the total-entropy changes (equivalence-values) for these transformations states:

> With regard to the magnitude of the equivalence-value it is at once seen that the value of a change from work into heat must be proportional to the quantity of heat generated, and that beyond this it can only depend on its temperature. We may therefore express

> generally the equivalence-value of the generation out of work of the quantity of heat Q, of temperature T, by the formula $Qxf(T)$, where $f(T)$ is a function of temperature which is the same in all cases. If Q is negative in this formula, what is expressed is that the quantity of heat Q has been transformed, not out of work into heat, but out of heat into work (1879, p. 98)

Eventually, Clausius was able to make the following identification $f(T) = 1/T$, with T being the absolute temperature (1879, pp. 100-109), in terms of which Clausius total-entropy changes for the transformation of work into heat, and vice versa, took the following form: Q/T, and $-Q/T$, respectively, expressions these that make evident the fact that for Clausius the transformation of heat into work, in contradiction with the notion enforced by the law of increasing entropy, is a negentropic process.

2) In our construction, as suggested by Figure 1, the self-organizing structure is work and engine at the same time. Even if the fluid in Rayleigh-Bénard convection supplies, in a way of speaking, the structure's building blocks; it is work however, in its capacity of chaos-ordering agent, the one playing the role of grout, stonemason, and structure's conservator in the way implied by Kant when stating that an organism is "*both cause and effect of itself*" (Schneider and Sagan, 2005, p. xii). In this perspective the self-organizing structure is the spatial manifestation of an energy-upgrading engine capable of outputting the amount of work required to maintain its structural integrity against the structure-breaking tendency of the same irreversible energy flows from which it feeds.

At the beginning of the path of increasing temperature gradients characterizing the temporal evolution of a given experimental *RBC*-cell the only process taking place is the conductive, irreversible, and entropic transfer of increasing amounts of heat. Along this path, a temperature gradient is reached at which the self-organization of the fluid into convection-cells or convection-rolls is witnessed, event marking the transition from conduction to structure-mediated convection as sole heat transfer mechanism. A good description of the structure's emergence can be found in the following words of Goldstein:

> The self-organizing patterns that emerge in the Bénard liquid indicate a radical change in the functioning of the system...This change is self-directed, self-generated, and self-guided as the system reconfigures

> its resources in the face of a far-from-equilibrium challenge, which in this particular case, is heat. (1994, p. 37)

This radical change brings forward the coexistence of two opposing effects: entropy production associated to the irreversible and now convective flow of heat between the source and its sink ($\Delta S_+ > 0$), and entropy reduction due to the architectural and functional order of the structure ($\Delta S_- < 0$); in simpler words, it brings forward the coexistence of entropy and negentropy. It is precisely in terms of these opposite effects that the radical change here taking place –the emergence of order out of chaos- finds expression in the form of a precise neutralization of the positive entropic contribution by an equal magnitude but oppositely signed negentropic contribution. Our contention is that no other result but a precise identity in magnitudes of these opposing factors can be as radical as the change experienced by the *RB*-cell at this critical point. In fact, no other result but that of a non-entropic universe i.e., $\Delta S_u(\text{critical point}) = \Delta S_+ + \Delta S_- = 0$ appears possible. It cannot keep evolving along an entropic path as this situation implies continuity, i.e., no radical change, no breaking away with the previous condition; it cannot be negative as it cannot transit from positive to negative without passing first through zero. If so, the emergence of the self-organizing structure transforms what previously was an entropic universe into a non-entropic one. *This is the negentropic conceptualization of Rayleigh-Bénard convection.* We can refer to the *RB*-cell as a universe because no other processes but the ones previously referred of heat transfer and structure permanence take place in it. From this point on the universe of *RBC* becomes the battleground of order vs. chaos. The structure will remain for as long as its own negentropy is capable of opposing the entropic effects of the irreversible energy transfer. If this autopoietic engine that is the structure can respond to higher irreversible throughputs of heat with equally higher outputs of ordered energy (work) then we will have permanence along a nonentropic path. If not, the structure will disappear and the universe will become, again, entropic. Even if not applicable to *RBC*, we might speculate that if the response consists of negentropic outputs in excess of those of an entropic nature then we would have evolution, and here the universe would be transiting from a nonentropic to a negentropic condition.

It needs to be noted that the inability, actually the factual impotence of the law of increasing entropy for producing rational, logically sound, and experimentally testable models of self-organization can be traced to its blind

and senseless negation of the negentropic effect associated to the transformation of heat into work. By doing this, thermodynamics threw away the only tool capable of making sense of self-organization.

8. The work-recycling mechanism and the terms in the equation for the total-entropy change of *RBC*

The equation for the total-entropy change (ΔS_u) of Rayleigh-Bénard convection, as it has been modeled in Figure 1, will be written, under the guidance of eq. (41) of Part I, as follows:

$$\Delta S_u = -\frac{Q_h}{T_h} + \frac{Q_h}{T_1} - \frac{w}{T_h} - \frac{Q_h}{T_1} + \frac{Q_h}{T_2} - \frac{w}{T_1} - \frac{Q_h}{T_2} + \frac{Q_h}{T_3} - \frac{w}{T_2} + \cdots - \frac{Q_h}{T_{n-1}} + \frac{Q_h}{T_c} - \frac{w}{T_{n-1}} \qquad (20)$$

In the previous equation T_h and T_c stand for either: the critical temperatures i.e., the ones defining the temperature gradient associated with the emergence of the convective structure, as well as for those temperatures associated with the structure's permanence beyond the critical point; Q_h represents the engines cyclical heat input, and $T_1, T_2, T_3 \ldots T_{n-1}$ the temperatures of the intermediate reservoirs as depicted in Figure 1.

As previously noted, the challenge faced by the convective structure in the form of increased entropy production due to the increased flow of heat associated to the larger temperature-gradients succeeding the so-called critical gradient, is met by the autopoietic coupling via the increased output of negentropy (work) associated also to the increased flows of heat.

That gradient at which ordered convection proves incapable of handling the increased heat flow rates associated to increasing gradients marks the start of the demise of the convective structure at the hands of the increased heat transfer capacity of turbulent convection.

Each of the engines of the coupling, it should be noted, contributes with three different entropy terms to equation (20). Thus, the first triad on the right hand side of this equation, namely $(-Q_h/T_h) + (Q_h/T_1) - w/T_h$ corresponds to engine 1 and its terms represent, in the order given, the entropy change of its hot reservoir of temperature T_h in releasing an amount of heat Q_h, the entropy change of its cold reservoir of temperature T_1 in receiving an amount of heat Q_h, with the third term representing the negentropy associated to the production of the amount of work w out of heat of temperature T_h [Part I of this series, eq. (33)]. In the same manner the second triad, namely

$(-Q_h/T_1) + (Q_h/T_2) - w/T_1$ corresponds to engine 2 and its terms represent, in the order given, the entropy change of its hot reservoir of temperature T_1 in releasing an amount of heat Q_h, the entropy change of its cold reservoir of temperature T_2 in receiving an amount of heat Q_h, and the negentropy associated to the production of the amount of work w out of heat of temperature T_1, and so on for the other triads. The presence of similar terms of opposite sign in engines 1 and 2, namely Q_h/T_1 and $-Q_h/T_1$ stems from the fact that the cold reservoir of engine 1 doubles as the hot reservoir of engine 2. This situation rules for all subsequent pairs of engines: $(2,3)$, $(3,4)$,...$(n-2, n-1)$ and $(n-1, n)$

In the understanding that providing an explanation for the nature of each of the terms of the triad of any given engine provides an explanation for all others is that we will center our attention on the representation given in Figure 2 for operation or cycle i of engine b working between a hot reservoir of temperature T_a and a cold one of temperature T_b. According to this, the previous and subsequent cycles or operations of this engine are, in this order, $i-1$, and $i+1$. Serves to keep in mind that $Q_h = Q_c + Q_i = Q_c + w_i$, with Q_c representing that portion of Q_h which is transferred by the engine from its hot to its cold reservoir, and Q_i representing the portion of Q_h which ends up being transformed into the equivalent amount of work w_i. With these considerations in place let us bring here the already noted fact that the coupled engines shown in Figure 1, even if optimally efficient, are irreversible (Íñiguez, 2020). The reason for this being that the work output of any of these optimally efficient engines is never available for purposes of recuperation of the initial condition. This work becomes temporarily unavailable when incorporated into the structure, and permanently unavailable when disincorporated in the form of an equivalent amount of heat at the temperature of the engine's cold reservoir.

In accord with these considerations, we can recognize the following changes taking place along cycle i of engine b:

1) The work w_i produced by this engine out of an equivalent amount of heat Q_i of temperature T_a, alongside its associated negentropy in the amount $-w_i/T_a$, pair which for economy of expression will be represented as $[w_i, -w_i/T_a]$, are incorporated by the structure. As this takes place an identical amount of work w_{i-1}, the product of engine's b previous cycle, as well as its associated negentropy in the amount $-w_{i-1}/T_a$, i.e., the pair $[w_{i-1}, -w_{i-1}/T_a]$, are disincorporated from it.

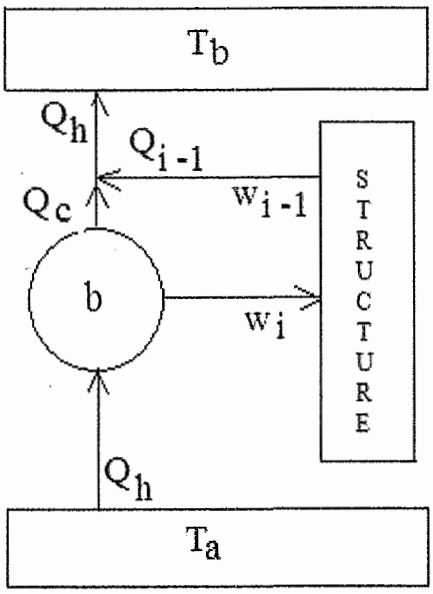


Figure 2. The work-recycling mechanism operating in the autopoietic model for Rayleigh-Bénard convection. The Figure above refers to cycle i of engine b. As the work w_i produced in this cycle is incorporated by the structure, an identical amount of work w_{i-1}, the one incorporated in the previous $i-1$ cycle, is disincorporated as the equivalent amount of heat Q_{i-1} and as such absorbed, alongside Q_c, by the cold reservoir of temperature T_b. Combined Q_c and Q_{i-1} produce Q_h. The fact that Q_h as well as Q_c remain unchanged for any cycle of any given engine of the coupling is the reason why no sub-index has been attached to them.

It serves to note here that the convective structure can be considered to be a mechanical reservoir under constant renovation by the expedient of an equally constant process of incorporation/disincorporation of [work/negentropy]. As such, the convective structure is nothing more than the temporal residence for the pair $[w_{i-1}, -w_{i-1}/T_a]$. This way, while w_{i-1} becomes part of the convective structure, $-w_{i-1}/T_a$ becomes part of the structure's negentropy. Such contribution comes to an end with the replacement of $[w_{i-1}, -w_{i-1}/T_a]$ by $[w_i, -w_i/T_a]$. At this point it is w_i the

one forming part of the structure and $-w_i/T_a$ part of the structure's negentropy.

Let us make clear here that when speaking about work and negentropy as a pair we do not mean to imply that they are two separate entities. Negentropy is inherent to the higher quality energy form we call "work", it is its essential characteristic.

2) Its mission complete, and its role taken over or assumed by w_i, w_{i-1} is rejected (degraded) by the structure via the process $w_{i-1} \rightarrow [Q_{i-1}(T_b)]$ and transferred as the equivalent amount of heat Q_{i-1} to the reservoir of temperature T_b. This energy-degrading process, which produces entropy in the amount of w_{i-1}/T_b [Part I, eq. (43)], in combination with the one through which w_{i-1} is generated in cycle $i-1$ out of the amount of heat Q_{i-1} of temperature T_a, define the following irreversible heat transfer which having started in cycle i -- 1, concludes in cycle i:

$$[Q_{i-1}(T_a)] \rightarrow w_{i-1} + w_{i-1} \rightarrow [Q_{i-1}(T_b)] = [Q_{i-1}(T_a)] \rightarrow [Q_{i-1}(T_b)]$$

The entropy change for this irreversible heat transfer is written below:

$$\Delta S\{[Q_{i-1}(T_a)] \rightarrow w_{i-1}\} + \Delta S\{w_{i-1} \rightarrow [Q_{i-1}(T_b)]\} = \Delta S\{[Q_{i-1}(T_a)] \rightarrow [Q_{i-1}(T_b)]\} = -w_{i-1}/T_a + w_{i-1}/T_b$$

Of the two terms composing this entropy change, while $-w_{i-1}/T_a$ forms part of the triad of cycle $i-1$, w_{i-1}/T_b contributes, as will be seen below, to the definition of the triad of cycle i.

4) Cycle i concludes with the absorption by the heat reservoir of temperature T_b of the heat Q_c passing through the engine, and Q_{i-1} coming from the dissipation of w_{i-1}. At this point the following four entropy changes have taken place in cycle i: $-Q_h/T_a$ associated to the hot reservoir releasing the amount of heat Q_h of temperature T_a; Q_c/T_b associated to the amount of heat Q_c transferred by the engine to the cold reservoir of temperature T_b; Q_{i-1}/T_b associated to process $w_{i-1} \rightarrow [Q_{i-1}(T_b)]$ through which the structure transfers to the reservoir of temperature T_b the heat Q_{i-1} produced by the disincorporation of w_{i-1}, and $-w_i/T_a$ associated to the production of w_i out of the amount of heat Q_i of temperature T_a. The pairing of Q_c/T_b with Q_{i-1}/T_b produces Q_h/T_b and in doing so reduces the number of entropy changes associated to cycle i of engine b to the triad $-Q_h/T_a$, Q_h/T_b, and $-w/T_a$. The sub-index of the work term has been eliminated on reason of the fact that each cycle of each and every engine of the coupling outputs the same amount of work, namely w. Following the results obtained for engine b, the

triad for engine c, the one which working between reservoirs of temperature T_b and $T_{c\prime}$ feeds from the heat Q_h transferred to the former reservoir by engine b, takes the following form $-Q_h/T_b + Q_h/T_{c\prime} - w/T_b$, and so on for the rest of the engines. Note that $T_{c\prime}$ was used in the previous equation in order to distinguish it from the coldest reservoir in Figure 2. The summation of all the triads produced by the transit of Q_h between the reservoirs of temperatures T_h and T_c is what eq. (20) represents. It is, in a way, the total-entropy change of a cycle of cycles.

Let us finally point out here the fact that the role played by w_{i-1} in cycle i, will be played by w_i in cycle $i + 1$ of engine b.

Figures 1 and 2, it has to be recognized, might convey the wrong idea that the coupling is something operating outside the convective structure when in reality it represents the structure as well as its subsumed maintenance mechanism: cause and effect at the same time. The identity existing between the magnitudes of the pairs [work, negentropy] coming in and going out of the structure indicates a condition of dynamic equilibrium, just like an organism in which anabolic build-up and catabolic break-down proceed at the same rate (von Bertalanffy, 1975, p. 47). The renovation/maintenance mechanism inherent to the model of Figure 1 identifies it as autopoietic in the way such a term has been defined by Jantsch (1980, p. 10)

> An autopoietic system is in the first line not concerned with the production of any output, but with its own self-renewal in the same process structure. Autopoiesis is an expression of the fundamental complementarity of structure and function, that flexibility and plasticity due to dynamic relations, through which self-organization becomes possible.

9. The final form of the total-entropy equation for Rayleigh-Bénard convection

The cancellation pattern evident in equation (20) leads to

$$\Delta S_u = -\frac{Q_h}{T_h} + \frac{Q_h}{T_c} - \frac{w}{T_h} - \frac{w}{T_1} - \frac{w}{T_2} - \frac{w}{T_3} \ldots - \frac{w}{T_{n-1}} \qquad (21)$$

Factoring w out of the work/temperature quotients of the previous equation gives us:

$$\Delta S_u = -\frac{Q_h}{T_h} + \frac{Q_h}{T_c} - w \sum_h^{n-1} \frac{1}{T_i} \qquad (22)$$

Obtaining a simple expression for ΔS_u depends on the evaluation of the summation shown above. This task starts by considering the graph of $1/T$ $vs.T$ from T_c to T_h. Let us mark on the x-axis, in addition to T_c and T_h, the corresponding values for T_{n-1} ... T_3, T_2, T_1. Looking back at Figure 1 we will be able to note in this regard that in the interval $[T_{n-1}, T_c]$ the extremes (endpoints) correspond, in the order written, to the temperatures of the hot and cold reservoirs of engine n; the same applies for interval $[T_2, T_3]$ for engine 3; to $[T_1,\ T_2]$ for engine 2, and $[T_h,\ T_1]$ for engine 1. If we pair the length of each of these intervals with a y-coordinate equal to the inverse of the temperature of its corresponding hot reservoir, we will get a number of rectangles equal to the number of coupled engines, with areas, in the order given, of magnitude $(T_{n-1} - T_c)/T_{n-1}$...$(T_2 - T_3)/T_2$, $(T_1 - T_2)/T_1$, $(T_h - T_1)/T_h$.

The summation of these areas produces the total area A given below:

$$A = \frac{T_h - T_1}{T_h} + \frac{T_1 - T_2}{T_1} + \frac{T_2 - T_3}{T_2} + \cdots + \frac{T_{n-1} - T_c}{T_{n-1}} \qquad (23)$$

It should be recognized that the right-hand side terms of equation (23) representing the areas of each of the n rectangles previously mentioned, are also representations of the efficiencies of the respective engines. The already noted fact that the efficiency of any engine is equal to that of any other allows us to assert that the area of any of the said rectangles is equal to that of any other. Let us also note that all the critical temperature data of Table 1, consisting in a number of pairs (T_h, T_c) corresponding to the temperatures of the hot reservoir (source) and cold reservoir (sink) at the moment in which organization emerges in Rayleigh-Bénard convection, data to which the equations to be here obtained are to be applied, produce $(1/T)$ $vs.T$ lines of practically zero slope. The application of equation $[(1/T_h) - (1/T_c)]/(T_h - T_c)$ to the data of Table 1 produced a slope of ≈ -0.00001 for each of the gradients there shown [Appendix 1]. This result, suggesting an approximate common height for the n-rectangles, in combination with their previously established identical areas, leads to the conclusion that the bases of these rectangles can also be taken as equal to one another, and also equal to $(T_h - T_c)/n$, i.e. $T_h - T_1 = T_1 - T_2 = \cdots = T_{n-1} - T_c = (T_h - T_c)/n$.

These considerations allow us to write equation (23) in the following form:

$$A = \frac{T_h - T_c}{n} \sum_h^{n-1} \frac{1}{T_i} \qquad (24)$$

Let us now agree that another measure of A can be obtained through the following equation

$$A = \int_{T_c}^{T_h} \frac{1}{T} dT = \ln \frac{T_h}{T_c} \qquad (25)$$

Equating now the previous two expressions for A leads us to

$$\frac{T_h - T_c}{n} \sum_h^{n-1} \frac{1}{T_i} = \ln \frac{T_h}{T_c} \qquad (26)$$

From which we get the desired result:

$$\sum_h^{n-1} \frac{1}{T_i} = \frac{n \ln (T_h/T_c)}{T_h - T_c} \qquad (27)$$

As discussed in Appendix 2, the precision of estimation, i.e., the closeness between the true results provided by $\sum_h^{n-1}(1/T_i)$ with their estimates provided by $[n \ln(T_h/T_c)]/(T_h - T_c)$ increases, with independence of the magnitude of the critical gradient, with increasing n. Thus, for critical temperatures of $T_h = 300\text{K}$ and $T_c = 297\text{K}$, or equivalently, for a critical gradient of $\Delta T = 3\text{K}$, the estimation error is 0.17% for $n = 3$, 0.10% for $n = 5$, 0.05% for $n = 10$, and 0.03% for $n = 15$. As seen in the graph there included, the estimation error becomes arbitrarily small for arbitrarily large numbers of coupled engines.

Substitution of equation (27) in (22) leads to the following equation for the entropy change of the universe of Rayleigh-Bénard convection:

$$\Delta S_u = -\frac{Q_h}{T_h} + \frac{Q_h}{T_c} - \frac{n\, w \ln(T_h/T_c)}{T_h - T_c} \qquad (28)$$

If w in equation (28) is replaced in accord to $w = \eta\, Q_h$ and the resulting $n\,\eta$ product written, as allowed by equation (11), as η_{cc}, and if we further factor Q_h, we will get, after some algebra, the following re-expression of equation (28):

$$\frac{\Delta S_u}{Q_h} = \frac{T_h - T_c}{T_h T_c} - \frac{\eta_{cc} \ln (T_h/T_c)}{T_h - T_c} \qquad (29)$$

As expressed, equation (29) gives the entropy change for the universe of Rayleigh-Bénard convection per unit of heat fed to the coupling. For economy in notation this entropy change will be further represented as $\Delta S_u/Q_h = \Delta S'_u$

Now, for an arbitrarily large number of coupled engines η_{cc} in equation (29) can be replaced according to eq. (19):

$$\eta_{cc} = -\ln(1-\eta^*) = -\ln(T_c/T_h) = \ln(T_h/T_c) \qquad (30)$$

If in addition to this last result we introduce to equation (29) the previously mentioned notation change, we will get

$$\Delta S'_u = \frac{T_h - T_c}{T_h T_c} - \frac{[\ln(T_h/T_c)]^2}{T_h - T_c} \qquad (31)$$

This is the equation quantifying, per unit of Q_h, the entropy change of the universe of Rayleigh-Bénard convection as modeled in Figure 1. The first term of the right-hand side represents what we will call the entropic opposite (ΔS_+), i.e.

$$\Delta S_+ = (T_h - T_c)/(T_h T_c) \qquad (32)$$

The form of this term makes it easy to identify it as originating in the irreversible transfer of heat taking place between the source and the sink, fact that explains its always positive contribution. The last term on the right-hand side represents, on its part, what we will call the negentropic opposite (ΔS_-), i.e.

$$\Delta S_- = -[\ln(T_h/T_c)]^2/(T_h - T_c) \qquad (33)$$

This term, with an always negative contribution, represents the negentropy associated to the convective structure. It might be fair to say that this is very likely the first time that the negentropy associated to the structural and functional order in self-organizing phenomena has been quantified.

The previous identifications of the entropic and negentropic opposites allows equation (31) to be written as

$$\Delta S'_u = \Delta S_+ + \Delta S_- \qquad (34)$$

Along the conductive regime preceding the emergence of order at the critical point there is no presence of ΔS_- and on reason of this the entropy change of the conductive universe is positive. As organization emerges at the critical point, the negentropic term enters the scene opposing its negative contribution to the positive one of the entropic term. According to our hypothesis, at the critical point the equilibrated contributions of the entropic and negentropic effects produce a nonentropic universe, i.e., a condition defined by

$$\Delta S'_u = \Delta S_+ + \Delta S_- = 0 \qquad (35)$$

Also in accord with our conceptualization of this self-organizing phenomenon, we assert that the structure remains, and with it the nonentropic condition of its universe, as long as it is capable of opposing the increased entropy production associated to larger temperature gradients with an equally increased output of negentropy. The temperature-gradients interval in which this condition persists defines the nonentropic interval of permanence of the structure. The quantification of the amplitude of this nonentropic interval is the matter of the next section.

10. On the amplitude of the nonentropic interval

As noted above, the temperature gradients associated to the emergence and permanence of the convective structure are the ones defining what has been called the nonentropic interval. This means, if our notions are correct, that any such experimental gradient should produce $\Delta S'_u = 0$ when substituted in eq. (31). The quantification of this interval will start by writing eq. (31) in the following form:

$$\Delta S'_u = \frac{T_c[(T_h/T_c) - 1]}{T_h T_c} - \frac{[\ln(T_h/T_c)]^2}{T_c[(T_h/T_c) - 1]} \qquad (36)$$

In terms of the variable x, defined as $x = T_h/T_c$ the previous equation becomes:

$$\Delta S'_u \cdot T_c = \frac{x-1}{x} - \frac{(\ln x)^2}{x-1} \qquad (37)$$

In attention to the right-hand side of the previous equation, we will define the following functions of x:

$$f(x) = (x-1)/x \qquad (38)$$

$$g(x) = (\ln x)^2/(x-1) \qquad (39)$$

$$h(x) = f(x) - g(x) \qquad (40)$$

In terms of equations (38) and (39), equation (37) takes the following form:

$$\Delta S'_u \cdot T_c = f(x) - g(x) \qquad (41)$$

This way, the condition that our model associates with both emergence and permanence of the convective structure, namely $\Delta S_u = 0$, or equivalently $\Delta S'_u = 0$, becomes equivalent with $f(x) = g(x)$. For as long as this last condition is met, the path of *RBC* will be nonentropic.

As made evident by eq. (31), it is only when $T_h = T_c$, or equivalently, when $x = 1$, that $\Delta S'_u = 0$. For all $x > 1$ we will have $\Delta S'_u > 0$, an indication of the fact, evinced by Figures 3 and 4, that even if coincident at $x = 1$, the paths of $f(x)$ and $g(x)$ diverge, with $f(x) > g(x)$, as the value of x increases. Even so, as made apparent by Figure 4, there is an interval of values of x in which the difference between $f(x)$ and $g(x)$ is so small that they appear to overlap. This is the interval in which the green colored curve of Figure 4 appears to coincide with the x-axis.

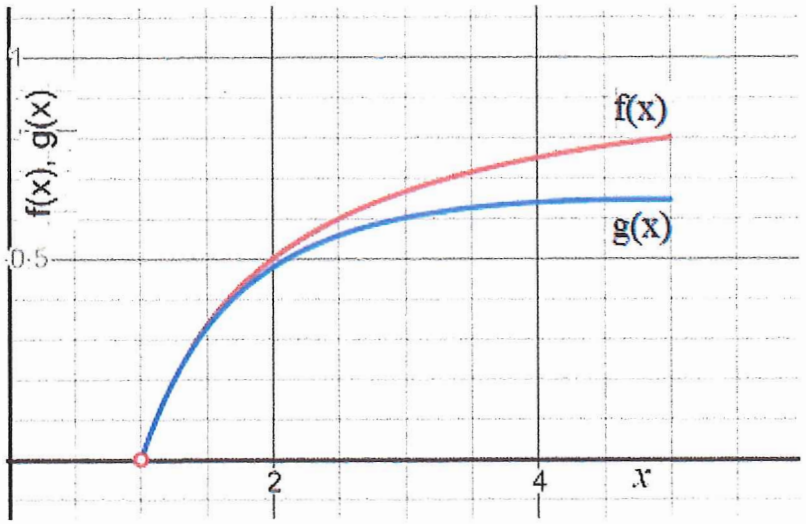


Fig. 3. Graphs of $f(x)$ and $g(x)$ as defined in equations (39) and (40), for $0 < x \leq 5$.

The facts that $f(x)$ refers to the positive entropic contribution coming from the irreversible heat flow from the source of heat to its sink, while $g(x)$ comes from the negentropy associated to the architectural and functional order of the convective structure, indicate that along this interval entropy production is at a minimum. It might very well be that subsumed by this region of minimum entropy production there exists an interval of values of x for which the difference between $f(x)$ and g(x) turns actually to be so small as to be negligible, in which case such an interval could be identified with what we have called the nonentropic interval.

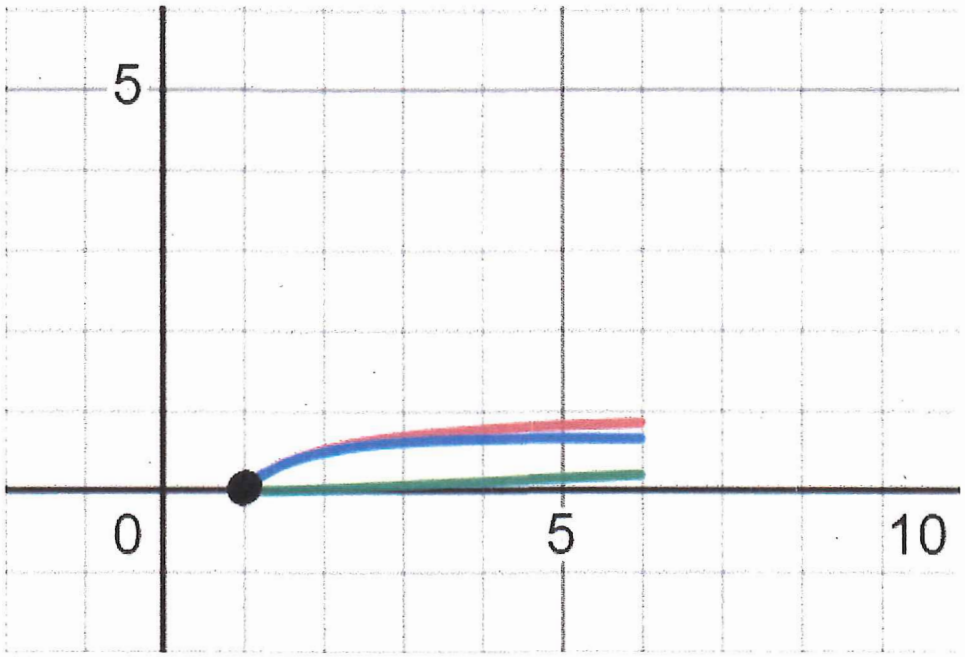


Figure 4. The figure shows the functions $f(x)$ in red, $g(x)$ in blue, and their difference $h(x) = f(x) - g(x)$ in green. Figures 3 and 4 were drawn using the software of desmos.com

The minimum entropy production characterizing this 'nonentropic' interval parallels the minimum entropy production characterizing reversible processes. *Stricto sensu* no process can be reversible on reason of the fact that their defining zero total-entropy condition can only be fulfilled by not occurring, or what is the same thing in other words, by zero-gradients.

In what is commonly called a reversible heat engine:

> (The) variable body, if placed in connection with a heat reservoir to receive or give off heat, must have the same temperature of the reservoir; for it is only in this case that the heat can pass as readily from the reservoir to the body as in the reverse direction, and if the process is reversible it is requisite that this should be the case. This condition cannot indeed be exactly fulfilled, since between equal temperatures there can in general be no passage of heat whatever; but we may at least assume it to be so nearly fulfilled that the small

> remaining differences of temperature may be neglected. (Clausius 1879, p. 106)

Let us agree, in this regard, that the irreversible transfer of a unit of heat between two heat reservoirs of temperatures T_h and T_c produces entropy in the amount of $\Delta S = (T_h - T_c)/(T_h T_c) = \Delta T/(T_h T_c)$. The so-called reversible transfer of this unit of heat, propelled by an infinitesimal gradient of temperature $dT = T_h - T_c$, produces, on its part, entropy in the amount of $dS = dT/(T_h T_c) = dT/(T_c^2 + T_c dT)$. The fact that $T_c^2 \gg T_c dT$ allows us to write $dS = dT/T_c^2$, from which we learn that $dS = 0$ if $dT = 0$. In other words, if $dT > 0$ so will be $dS > 0$. When the "...the small remaining differences of temperatures..." referred to by Clausius are neglected, so is the minute entropy change to they associated, namely $dS > 0$. In the ultimate analysis, the path of a reversible process is a path of minimum entropy production.

Transiting along paths of minimum entropy production appears not to be the exclusive province of the reversible processes of classical thermodynamics. In Rayleigh-Bénard convection we have a real process taking place, according to our model, along such a path. And what is, the reader might ask, the connection between this irreversible process and what thermodynamics classifies as reversible processes? The connection is work; efficient work production is the hallmark of what thermodynamics classifies as reversible processes; efficient work production and its efficient use is what characterizes the autopoietic coupling of optimally efficient heat engines with which we are modelling the behavior of *RBC*. It should be kept in mind that while the coupled engines are optimally efficient in that they output work at Carnot's efficiency, they are however irreversible if attention is given to the fact that their work output is never available for restoration of the initial condition (Iniguez 2020)

Our model, it needs to be recognized, provides no indication as to the amplitude of the nonentropic interval. The fact that the lower endpoint of this open interval is necessarily $x = 1$ means that the determination of its amplitude reduces to finding its upper endpoint. Doing so constitutes a test in itself of the model as the upper endpoint needs to be close enough to $x = 1$ to assure differences between $f(x)$ and $g(x)$ small enough to be deemed negligible, and while doing this, still agree with the experimental temperature gradients associated to the point at which the role of ordered-convection as

sole heat transfer mechanism begins to be challenged or replaced by turbulent convection.

x	$f(x)$ $\cdot 10^2$	$g(x)$ $\cdot 10^2$	$h(x) \cdot 10^2$
1.0010	0.099001	0.099001	0
1.0050	0.49751	0.49751	0
1.0100	0.99010	0.99010	0
1.0150	1.4778	1.4778	0
1.0160	1.5748	1.5748	0
1.0170	1.6716	1.6715	0.0001
1.0180	1.7682	1.7681	0.0001
1.0190	1.8646	1.8645	0.0001
1.0200	1.9608	1.9607	0.0001
1.0250	2.4390	2.4389	0.0001
1.0300	2.9126	2.9126	0.0001
1.0400	3.8462	3.8457	0.0005
1.0500	4.7619	4.7610	0.0009

Table 1. The calculation of $h(x) = f(x) - g(x)$ identifies $x = 1.0170$ as right-hand side bound of the nonentropic interval.

The identification of the upper end of the interval required a trial-and-error procedure which started by making a decision about the magnitude of the difference between $f(x)$ and $g(x)$ that could be considered small enough to be neglected. The procedure, illustrated with our final choice of 1 part in 1 million parts, or $h(x) = f(x) - g(x) = 0.000001$, consisted in the substitution in equations (39) and (40) of the values of x shown in the first

column of Table 1 in order to produce the numbers in columns 2 and 3. The differences between $f(x)$ and $g(x)$ for each of these numbers were calculated and tabulated as $h(x)$ in ascending order of magnitudes of x. The number marking the transition from a difference of zero to a difference of 0.000001 (or as $0.0001 \cdot 10^{-2}$) was the value of x selected as upper endpoint, which as the table indicates, was $x = 1.017$, and with it the nonentropic interval became $1 < x < 1.017$

Recognition of the fact that $f(x)$ and $g(x)$ are magnitudes respectively proportional, but not equal to the entropic and negentropic changes appearing in equation (31), requires of us to translate the criterion referred to above, namely i.e., $f(x) - g(x) = 0.000001$, into a difference of the form $\Delta S_+ - \Delta S_-$. These last magnitudes, respectively defined by equations (32) and (33), find themselves connected to $f(x)$ and $g(x)$, via eq. (37), in the following form $f(x) = \Delta S_+ \cdot T_c$, and $g(x) = \Delta S_- \cdot T_c$, in which case the said criterion will take the following form: $\Delta S_+ - \Delta S_- = [f(x) - g(x)]/T_c = 0.000001/T_c$. If T_c is assumed to be 300 K, and $1/300$ approximated to $1 \cdot 10^{-3}$, then in terms of entropy the criterion of 'negligibility' turns out to be $\Delta S_+ - \Delta S_- = 1 \cdot 10^{-9}$ or 1 part in 1 billion parts. A negligible difference indeed.

11. The Application of our model's results to experimental data

Two are the testable notions coming out of our model for *RBC*: 1) Any *experimental* critical gradient (the one at which the structure emerges), expressed as $x_e = (T_h/T_c)_{emergence}$ is expected to fall in the interval $1 < x < 1.017$, and 2) The *experimental* temperature gradient marking the transition from ordered-convection to turbulence, expressed as $x_d = (T_h/T_c)_{demise}$ is expected to coincide with the upper endpoint of the nonentropic interval, namely $x = 1.017$

The test of the first of these two notions will make use of the data produced by the experiments of Schmidt and Milverton (1935), and Schmidt and Saunders (1938), for the critical temperature gradient of water, that is, for the temperature gradient at the moment of emergence of Rayleigh-Bénard convective structure.

The data of Schmidt and Milverton (first two rows, columns 2 to 5), and Schmidt and Saunders (first two rows, columns 6 and 7) appear in Table 2. The third row gives the values of the experimental quotient x_e as defined above. The fact that each and every one of these experimental numbers

comply with $1 < x_e < 1.0170$ confirms the first of the two testable notions described above. The non-entropic nature of the universe associated to these gradients finds confirmation with the numbers given in rows 4, 5, 6, and 7 for $f(x)$, $g(x)$, $h(x)$, and $\Delta S'_u$.

T_h, K	298.05	296.90	294.00	291.62	291.50	292.95
T_c, K	296.35	295.90	292.95	290.78	290.25	290.85
x_e	1.0060	1.0030	1.0040	1.0030	1.0040	1.0070
$f(x) \cdot 10^2$	0.59642	0.29910	0.39841	0.29910	0.39841	0.69513
$g(x) \cdot 10^2$	0.59642	0.29910	0.39841	0.29910	0.39841	0.69513
$h(x) = f(x) - g(x)$	0	0	0	0	0	0
$\Delta S'_u = h(x)/T_c$	0	0	0	0	0	0

Table 2. The critical temperature data of Schmidt and Milverton, Schmidt and Saunders, and the results obtained when substituted in equations (39), (40), and (41) are shown above.

In addition to the critical temperature gradients, Schmidt and Saunders experiments also produced temperature data beyond the point of emergence:

> In some previous experiments (Schmidt and Milverton, 1935) a layer of water between two horizontal plates was slowly heated from below. The critical temperature difference at which the water began to move was found from a change in the slope of the curve relating the difference of temperature between the plates and the rate of supply of heat to the lower plate...The present experiments were undertaken to find whether any change in the type of motion occurs at higher temperature differences...The results appear to indicate a change in the type of motion at a critical value of λ equal to about 45,000... the steady cellular motion which occurs at lower values of λ, breaks down at about $\lambda = 45{,}000$ and turbulent motion begins, in which the water temperature fluctuates with time (pp. 216, 221).

As to the nature of the transition, they assert that

> If a horizontal layer of fluid, initially at rest, is heated from below, instability and cellular motion set in when λ ... is about 1700. (Our) experiments show that on further heating the layer becomes completely turbulent at about $\lambda = 45{,}000$ (p. 228).

In the previous quotes λ represents the Rayleigh number. The data for the temperature gradients associated to this transition appear in the first two rows of Table 3. They were read from these author's Figure 2 in page 220.

T_h, K	294.90	295.85	297.15	297.30
T_c, K	292.50	292.75	293.15	293.10
x_d	1.0082	1.0106	1.0136	1.0143
x_{NT}	1.0170	1.0170	1.0170	1.0170
$f(x) \cdot 10^2$	0.81333	1.0489	1.34175	1.4098
$g(x) \cdot 10^2$	0.81333	1.0489	1.34173	1.4098
$h(x) = f(x) - g(x)$	0	0	0	0
$\Delta S'_u = h(x)/T_c$	0	0	0	0

Table 3. Schmidt and Saunders temperatures for the transition convection-turbulence given in the first two rows of Table 3 were re-expressed as x_d in the third row. These values are to be compared with the numbers given in row 4 for x_{NT} (negentropic thermodynamics' prediction) representing the upper endpoint of the nonentropic interval.

The comparison of the experimental values of Schmidt and Saunders for the quotient of the experimental temperatures associated to the point of emergence of turbulent convection, given in the third row of table 3 as x_d, with the upper limit of the nonentropic interval appearing in row 4 as $x_{NT.}$ evince the closeness of our model's prediction with the experimental values. All the differences between these values are smaller than 1%. The fact that all the experimental values fall within the nonentropic interval finds

correspondence with the numbers provided in the last four rows for $f(x)$, $g(x)$, $h(x)$, and $\Delta S'_u$.

12. Conclusions

The agreement evinced by Tables 2 and 3 between the testable notions of our model and the experimental data of Schmidt and Milverton, and Schmidt and Saunders represent a first validation of our model as well as for the notion at its core, namely the negentropic nature of the energy-upgrading process represented by the transformation of heat into work, a notion radically departing from the accepted dogmas of the law of increasing entropy.

Further validation or plain rejection of the model and its results will require continued contrast with experiment.

Dedication and acknowledgement

For Prof. Alejandro Dueñas Duran (†), Prof. Juan Buendia (†), Dr. Douglas M. McEachern, Jr. (†), Dr. Pedro Lehmann (†), Dr. Pedro Joseph-Nathan, Dr. Josef E. Herz (†), and Dr. Witold Brostow.

The comments provided by Dr. Sebastiano Correra in regard to paper #2 were instrumental in keeping some of my discussions within the bounds of logic and reason.

References

Anton, H. (1999) Calculus. Wiley, NY

Bénard, H. (1900) Les Tourbillons cellulaires dans une nappe liquid, *Revue Générale des Sciences pures et appliquées*,11, pp. 1261-1271 and 1309-1328

Bénard, H. (1901) Les Tourbillons cellulaires dans une nappe liquid transportant de la chaleur par convection en régime permanent, *Annales de Chimie et de Physique*, 23, pp. 62-144

Carnot, S. (1824/1960) *Reflections on the Motive Power of Fire*. Dover, NY

Chandrasekhar, S. (1981) *Hydrodynamic and Hydromagnetic Stability*, Dover, NY

Clausius, R. (1879) *The Mechanical Theory of Heat*. Macmillan, London.

Epsiloney, Mathematics Stack Exchange 2015:

https://math.stackexchange.com/questions/1398696/why-does-lh%C3%B4pitals-rule-work-for-sequences

Feistel, R. and Ebeling, W. (1989) *Selforganization, Entropy and Development*. Kluwer, Dordrecht

Goldstein, J. (1994) *The Unshackled Organization*. Productivity Press, Portland.

Haxton, B. (2003) *Heraclitus*. Penguin, NY

Non-linear Physics Group of the University of Toronto: https://www.physics.utoronto.ca/~nonlin/thermal.html

Íñiguez, J. C. (2020) An Irreversible Engine Working at the Reversible Efficiency, *International Journal of Chemistry* **12**, 1-5: https://doi.org/10.5539/ijc.v12n2p1

Íñiguez, J. C. (2010) An Unusually Efficient Coupling of Heat Engines, *Apeiron*, **17**, 150-172: http://redshift.vif.com/JournalFiles/V17NO2PDF/V17N2INI.pdf

Jantsch, E. (1980) *The Self-Organizing Universe*. Pergamon, Oxford

Lord Rayleigh (1916) On convective currents in a horizontal layer of fluid when the higher temperature is on the underside, *Phil. Mag.*, 32, pp. 529-546

Schmidt, R. J. and Milverton, S. W. (1935) On the Instability of a Fluid when Heated from Below. *Proceedings of the. Royal Society* of *London, Series A*, **152**, 586-594, https://doi.org/10.1098/rspa.1935.0209

Schmidt, R. J and Saunders, O. A. (1938) On the Motion of a Fluid Heated from Below, *Proceedings of the. Royal Society* of *London, Series A*, **165**, 216-228, https://doi/10.1098/rspa.1938.0055

Schneider, E. D. and Sagan, D. (2005) *Into the Cool: Energy Flow, Thermodynamics, and Life*. The University of Chicago Press, Chicago.

Soltzberg, L. J. (1989) Self-organization in Chemistry: The Larger Context, *Journal of Chemical Education,* **66**, 187, https://doi.org/10.1021/ed066p187

Von Bertalanffy, L. V. (1975) *Perspectives on General System Theory*. George Braziller, New York.

Appendix 1

In the table below T_c and T_h stand for the critical temperatures corresponding to the experiments of Schmidt and Milverton, and Schmidt and Saunders used in the arguments of the previous sections. The slopes of the lines connecting the points $(T_c,\ 1/T_c)$ and $(T_h,\ 1/T_h)$ from they obtained, have been annotated in the last row in Table A1. The largest slope of the set is $\approx$ 0.000012

T_h, K	298.05	296.90	294.00	291.62	291.50	292.95
T_c, K	296.35	295.90	292.95	290.78	290.25	290.85
ΔT	1.70	1.00	1.05	0.84	1.25	2.1
$(1/T_h)\cdot 10^3$	3.3551	3.3681	3.4014	3.4291	3.4305	3.4135
$(1/T_c)\cdot 10^3$	3.3744	3.3795	3.4135	3.4390	3.4453	3.4382
$\Delta(1/T)\cdot 10^6$	19.3	11.4	12.1	9.9	14.8	24.7
$[\Delta(1/T)/\Delta T]\cdot 10^6$	11.3	11.4	11.5	11.8	11.8	11.8

Table A1. Slopes of the lines connecting critical temperature points $(T_c,\ 1/T_c)$ and $(T_h,\ 1/T_h)$.

Appendix 2

To ascertain the error incurred when the true value of the summation $\sum 1/T_i$ is evaluated with its approximation $n[\ln(T_h/T_c)]/(T_h - T_c)$ we quantified both of these expressions with $T_h = 300\text{K}$, $T_c = 297\text{K}$, and four different values of n: 3, 5, 10, and 15. The previous temperatures were selected for this exercise on reason of their similarity with the experimental temperatures of Table A1. In each case we report the temperatures of the reservoirs as well as the value of the summation $\sum_h^{n-1} 1/T_i$. This value is then compared with that obtained from $[\ln(T_h/T_c)]/(T_h - T_c)$, and their difference (the error of estimation) is reported in % units.

For the critical temperatures $T_h = 300\text{K}$, $T_c = 297\text{K}$, and $n = 3, 5, 10, 15$ the respective efficiencies calculated with equation (7) amount to 0.003344506, 0.002008048, 0.001004528, and 0.000669797. With this data at hand, the temperatures of the intermediate reservoirs were in each case calculated with equation (5). The results are given below. All the temperatures are in Kelvins.

For $n = 3$, $T_1 = 298.996648$, $T_2 = 297.9966521$ we get

$\sum_h^2 1/T_i = (1/T_h) + (1/T_1) + (1/T_2) = 0.010033594$

$n[\ln(T_h/T_c)]/(T_h - T_c) = 0.010050335$

$\%\text{Error (E)} = [(0.010050335 - 0.010033594)/0.010033594] \cdot 100$
$= 0.17$

For $n = 5$, $T_1 = 299.3975855$, $T_2 = 298.7963803$, $T_3 = 298.1963825$, $T_4 = 297.5975896$

$\sum_h^4 1/T_i = 0.016733869$, $n[\ln(T_h/T_c)]/(T_h - T_c) = 0.016750559$, $\%E = 0.10$

For $n = 10$, $T_1 = 299.6986414$, $T_2 = 299.3975853$, $T_3 = 299.0968318$, $T_4 = 298.7963803$,

$T_5 = 298.4962307$, $T_6 = 298.1963826$, $T_7 = 297.8968357$, $T_8 = 297.5975897$, $T_9 = 297.2986442$,

$\sum_h^9 1/T_i = 0.033484565$, $n[\ln(T_h/T_c)]/(T_h - T_c) = 0.033501119$, $\%E = 0.05$

For $n = 15$, $T_1 = 299.7990606$, $T_2 = 299.5982558$, $T_3 = 299.3975855$, $T_4 = 299.1970496$,

$T_5 = 298.996648$, $T_6 = 298.7963806$, $T_7 = 298.5962474$, $T_8 = 298.3962482$, $T_9 = 298.196383$,

$T_{10} = 297.9966517$, $T_{11} = 297.7970541$, $T_{12} = 297.5975903$, $T_{13} = 297.39826$, $T_{14} = 297.1990632$

$\sum_h^{14} 1/T_i = 0.050235262$, $n[\ln(T_h/T_c)]/(T_h - T_c) = 0.050251679$, $\%E = 0.03$

Summary of results in the form $(n, \%E)$: $(3, 0.17), (5, 0.10), (10, 0.05), (15, 0.03)$.

These calculations were repeated for $T_h = 301.2K$, $T_c = 297\text{K}$, and $T_h = 303\text{K}$, $T_c = 297K$. The respective results, in the same format, are: $(3, 0.23)$, $(5, 0.14)$, $(10, 0.07)$, $(15, 0.045)$ and $(3, 0.33)$, $(5, 0.197)$, $(10, 0.097)$, $(15, 0.063)$.

The graphs for these three sets of points appear in Figure A2, from which it can be seen that as n gets arbitrarily large, E gets arbitrarily small.

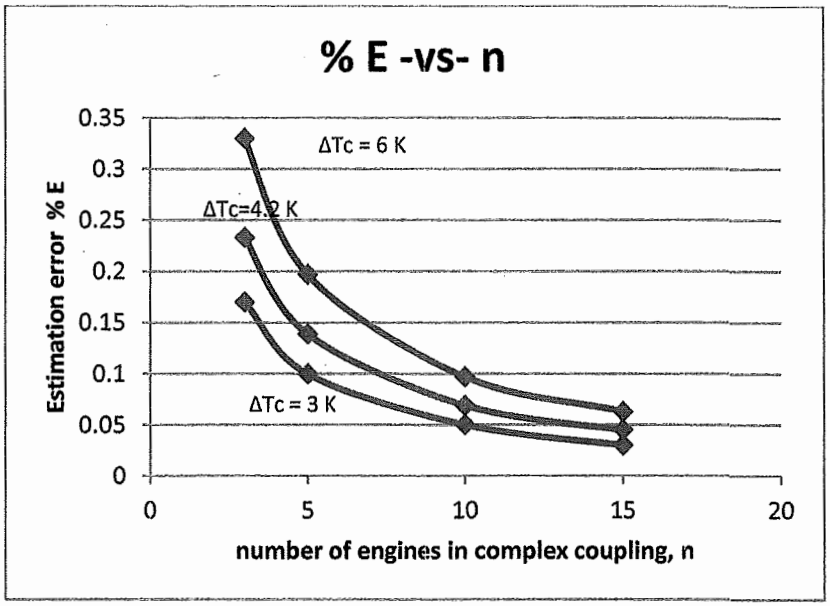


Figure A2. The estimation error $\%\, E$ measuring the difference between the true value of $\sum_h^{n-1} 1/T_i$ and its approximation given by $n[\ln(T_h/T_c)]/(T_h - T_c)$ is here graphed for three different critical gradients as a function of the number of engines n coupled

HADRONIC JOURNAL 45, 333 - 355 (2022)
DOI 10.29083/HJ.45.03.2022/SC333

HADRONIC CONDUCTIVITY: A NEW FORM OF SUPERCONDUCTIVITY

Alexander O.E. Animalu
Department of Physics and Astronomy
University of Nigeria, Nsukka
Enugu State, Nigeria
nascience@aol.com

Received October 20, 2021
Revised May 3, 2022

Abstract

Recent mathematical, theoretical and experimental studies have confirmed the exact validity of quantum mechanics for point-like particles in vacuum under linear, local and potential interactions, as occurring for electromagnetic interactions. The same studies have established a progressive generalization of Heisenberg's uncertainties according to the 1935 Einstein-Podolsky-Rosen (EPR) argument, with a corresponding recovering of Einstein's determinism for extended particles in deep mutual entanglement with ensuing non-linear, non-local and non-potential interactions, as occurring for extended hadrons under strong interactions. In this paper we indicate, one of novel advances in condensed matter physics and chemistry that are expected from the "completion" of quantum into hadronic mechanics according to the EPR argument. In particular, we present an apparently new form of superconductivity, here submitted for the first time under the name of "Hadronic Conductivity" consisting in the propagation of electron pairs (rather than individual electrons) in singlet coupling with "attractive" EPR entanglement and ensuring the Meissner effect (null magnetic field with consequential reduction of the resistance). Specific example of two molecules ($2H_2O$) "water current" as "hydroxil ion" consisting of oxonium (H_3O^+) and hydroxide (OH^-) states as "oxonium cloud" and "hydroxide biogas" is given.

KEY WORDS: EPR arguments with respect to "determinism" in quantum mechanics, condensed matter physics and chemistry and the "completion" in "hadronic" mechanics; Pairing in 2N-body(N>2) quantum electrodynamics of entangled "extended" current carriers in superconductivity of simple(metals) and complex systems(quasi-crystalline solids) vis-à-vis gas/liquid/solid states of (H_2O).

Copyright © 2022 by Hadronic Press, Inc., Palm Harbor, FL 34684, USA

OUTLINE

1. INTRODUCTION AND REVIEW
2. BROADENING OF DUAL DICHOTOMY TO TRICHOTOMY
3. CORRELATED DEFINITIONS OF HADRONIC CONDUCTIVITY AND SUPERCONDUCTIVITY
4. USE OF EPR ARGUMENT FOR RELATING HADRONIC CONDUCTIVITY AND SUPERCONDUCTIVITY
5. DISCUSSION & CONCLUSION
6. REFERENCES

1. INTRODUCTION AND REVIEW

Despite advances with Unitary Scattering Theory of Point-Particles in Conventional Quantum Mechanics, a number of fundamental problems have long been identified & partially tackled:

- Replacement of Infinities in quantum scattering formalism *ab initio* by convergence (Dirac 1981 [1]) & tackling imaginary potentials in Dissipative Nuclear Models by non-unitary theory.
- Need to replace Linear Character of quantum mechanics by Nonlinear character of nature (W. Heisenberg cited by R.M. Santilli 1978 [2])
- Einstein, Podolsky and Rosen (EPR) 1935[3] expression of doubts about "lack of completion of quantum mechanics" which underscores the motivation for this paper.
- Suggestions by R.M. Santilli 1978 [2] for construction of Non-unitary Covering of Quantum Mechanics under the name Hadronic Mechanics (HM)
- E. Recami [4]2005 Multi-verses, Micro-universes and Elementary Particles (Hadrons) (Fig.1) arXiv:hep-th/0505190 v1 21 May 20051978
- Non-unitary Lie-isotopic & Lie-admissible scattering theory of HM (Santilli &Animalu 2011[5])
- How to eliminate fractional baryon number and electric charge of SU(3) quarks by non-unitary transformation into integral lepton number and electric charge in O(4,2) current algebra (Animalu, 1972 [6]) & formulation of O(4,2)xSU(3)xU(1) lattice gauge group that decomposes to SO(10) approach to strong coupling ("holographic") superconductivity under the name geno-superconductivity in order to incorporate geno-Bragg's law obeyed by quasicrystal structures (Animalu, Nwakanma & Akpojotor 2013 [7] Afr. J. Physics Vol. 6, 1-18,(2013)).
- Correlated Current loops and Chandra Varma's Current Loop Model of High-Tc superconductivity & Akpojotor's superexchange pairing mechanism (in various articles by Animalu, Akpojotor, T.L.Gill ,[8] 2016 in African Journal of Physics Vol.9, p.10-59 (2016)).
- A.O.E. Animalu 2019 [9] -Lie-Admissible Approach to "Extended Relativity": Non-Linear Velocity, Mass and Charge Transformations in African *Journal of Phys. Vol.12, p.2-27 (2019) [reformatted from Hadronic J. Vol.10, 321-330(1987]*

To further underscore the question that led to the above scenario of cube-hexagon (geno-dual) metric tensor principle in three-dimensional projective geometry in the era of search for a unified field theory of gravity and electromagnetism we recall Animalu's interaction in 1968 with P.A.M. Dirac 1970[10] while he was searching for a Maxwell-like (dual) gauge principle to relate electric charge and magnetic charge in projective geometric terms. Following treatment of the Lorentz force and the linear Dirac (negative) energy relativistic wave equation for electric charge on the same footing as a corresponding dual of Lorentz force and a positive energy relativistic wave equation for magnetic charge and non-negative mass (represented by a current loop). Dirac's way of thinking led (as summarized below) to a correspondence principle

points↔particles, lines↔fields and planes↔currents

Such a correspondence principle implies geometric characterization of conventional electrodynamics and an analogous so-called e-magnetodynamics of current loop and to the Feynman space-time diagrammatic approach based on 10x10 representations of the dynamical group unifying strong interaction with electromagnetism and space-time geometry and violations of the discrete symmetries – parity(P), charge(C) conjugation, time-reversal(T) and spin-parity $SU^{>}(3)\times SU^{<}(3)\times U^{>}(1)U^{<}(1)\times O^{>}(4,2)\times O^{<}(4,2)$

This defined our SO(2N)xSO(2N) group-theoretic approach to scattering of N-particle (N=5) systems presented[11] in http://doi.org/10.52202/059404-0001 which we shall elaborate and distinguish from the "trichotomy" of Minkowski space of "extended relativity" R3xSO(3) and U(2)xSU(2) of elementary particles in Dirac's 1971 [1] SO(2N) second quantization scheme for condensed matter physics and chemistry with SO(10) linguistic geometry of two liquid water[12] molecules ($2H_2O$) as reactant "hydroxil ion" in oxonium(H_3O^+) + hydroxide(OH^-) states.

Due to the dimension and diversification of the existing literature in the EPR argument, prior this paper, interested colleagues should view the following:
Nine minutes video on EPR, verifications,
http://www.world-lecture-series.org/legacy-of-einstein-for-new-clean-energies
Tutoring Lecture I: Isomathematics
http://www.world-lecture-series.org/santilli-tutoring-i
Tutoring Lecture II: Verifications of the EPR argument
http://www.world-lecture-series.org/santilli-tutoring-ii
Primary references on recent verifications of the EPR argument
http://www.santilli-foundation.org/docs/references-epr-verifications.pdf

As a prelude to the importance of the EPR argument in this paper, Animalu recalls his visit to Brazil (1994), and being struck by E. Recami's emphases of how three colors of elementary particles could be captured in a "trichotomy" of Minkowski space of "extended relativity" R3xSO(3) and U(2)xSU(2) of elementary particles in Dirac's SO(2N) second quantization scheme for baryons and mesons interacting with Maxwell-Dirac 2N-dimensional gauge fields lead to SO(2N) = SO(10). And following his 2005 visit to Copenhagen via Einstein's monument at Switzerland, he was led to linguistic projective geometry of signs

and symbols which motivated his 21st C visits to India. China and Moscow and called for reformatting of his Lie-Admissible Approach to "Extended Relativity": Non-Linear Velocity, Mass and Charge Transformations in *African Journal of Phys. Vol.12, p.2-27 (2019) [reformatted from Hadronic J. Vol.10, 321-330(1987).* We shall therefore, begin with linguistic analogue/digital geometric SO (10) characterization of hadronic conductivity and superconductivity and "trichotomy" of the concepts of iso-electronium and iso-electronia in the physics of this apparently new state of condensed matter characterized by Geno-Bragg's Law for (quasicrystalline) Ca13Cd76 in 2012 article by Animalu [13] entitled "8x8 and 10x10 Hyperspace Representations of SU(3) and 10-fold Point- Symmetry Group of Quasicrystals"
https://www.researchgate.net/publication/258601350
as in hyperlink model of the H-atom.

To derive an appropriate generalization of Maxwell's classical electrodynamics **we introduce a six-vector (J,B) system which may be represented by 4x4 antisymmetric tensor analogous to the six-vector(E,B) system of Maxwell's electromagnetic field as follows:**

$$\|G_{\mu\nu}\| \equiv \begin{pmatrix} 0 & J_1 & J_2 & J_3 \\ -J_1 & 0 & B_3 & -B_2 \\ -J_2 & -B_3 & 0 & B_1 \\ -J_3 & B_2 & -B_1 & 0 \end{pmatrix} \sim \|F_{\mu\nu}\| \equiv \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix} \quad \textbf{(1)}$$

Now, the usual Maxwell's equation which are 3-vector equations, relating the electric field, E, magnetic field, B, and a conserved electric current $\mathbf{J^e}$, when rewritten iin 4-vector notations takes the form

$$\partial^t F_{\mu\nu} = J^e_\mu \qquad \textbf{(2)}$$

and has conventional dual symmetry with respect to interchange of electric and magnetic quantities, ($\mathbf{E \to B,\ B \to -E.\ J^e \to J^m}$) where $\mathbf{J^m}$ is the magnetic analog of the electric current . while the usual Lorentz force takes the form

$$\mathbf{f}^e = e\mathbf{E} + \mathbf{J}^e \times \mathbf{B}. \qquad \textbf{(3)}$$

Finally, to include the effect of gravity, we incorporate the space-time metric and curvature, by introducing (in addition to Eq.(2)) the determinant equation[3]

$$\mathrm{Det}\|F_{\mu\nu} - \lambda\eta_{\mu\nu}\| \equiv \lambda^4 - (R_{\mu\nu\rho\sigma}F^{\mu\rho}F^{\nu\sigma})\lambda^2 + (\in_{\mu\nu\rho\sigma} F^{\mu\rho}F^{\nu\sigma})^2 = 0, \qquad \textbf{(4)}$$

where , with $\|\eta_{\mu\nu}\| \equiv diag(+1,-1,-1,-1), R_{\mu\nu\rho\sigma} \equiv (\eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\rho}\eta_{\sigma\nu})$, **we have**

$$R_{\mu\nu\rho\sigma}F^{\mu\rho}F^{\nu\sigma} - 2 \in_{\mu\nu\rho\sigma} F^{\mu\rho}F^{\nu\sigma} = \begin{cases} \mathbf{E}^2 \\ \mathbf{B}^2 \end{cases} \Rightarrow \begin{cases} \mathbf{E}^2 - \mathbf{B}^2 \pm 2\mathbf{B}.\mathbf{E} = 0 \\ \mathbf{B}^2 - \mathbf{E}^2 \pm 2\mathbf{E}.\mathbf{B} = 0 \end{cases} \qquad (5)$$

For the corresponding magnetic pole array, we have the analog of Eq.(4),

$$\mathrm{Det}\|G_{\mu\nu} - \lambda\eta_{\mu\nu}\| \equiv \lambda^4 - (R_{\mu\nu\rho\sigma}G^{\mu\rho}G^{\nu\sigma})\lambda^2 + (\in_{\mu\nu\rho\sigma} G^{\mu\rho}G^{\nu\sigma})^2 = 0, \quad \textbf{(6)}$$

where

$$R_{\mu\nu\rho\sigma}G^{\mu\rho}G^{\nu\sigma} - 2 \in_{\mu\nu\rho\sigma} G^{\mu\rho}G^{\nu\sigma} = \begin{cases} \mathbf{J}^{m2} \\ B^2 \end{cases} \Rightarrow \begin{cases} \mathbf{J}^{m2} - \mathbf{B}^2 \pm 2\mathbf{B}.\mathbf{J}^m = 0 \\ \mathbf{B}^2 - \mathbf{J}^{m2} \pm 2\mathbf{J}^m.\mathbf{B} = 0 \end{cases} \qquad (7)$$

under the dual transformation ($\mathbf{J^m \to B,\ B \to -J^m}$).This result is summarized below.

Conventional Electrodynamics | **E-magnetodynamics of Current Loop**

$$\|F_{\mu\nu}\| \equiv \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix} \quad \| G_{\mu\nu} \| \equiv \begin{pmatrix} 0 & J_1 & J_2 & J_3 \\ -J_1 & 0 & B_3 & -B_2 \\ -J_2 & -B_3 & 0 & B_1 \\ -J_3 & B_2 & -B_1 & 0 \end{pmatrix}$$

Maxwell's eqn & Lorentz force

$$\partial^{\nu} F_{\mu\nu} = J^{e}_{\mu}; \mathbf{f}^{e} = e\mathbf{E} + \mathbf{J}^{e} \times \mathbf{B}.$$

$$\mathbf{E} \to \mathbf{B},\ \mathbf{B} \to -\mathbf{E}.\ \mathbf{J}^{e} \to \mathbf{J}^{m}$$

$$(\mathbf{J}^{m} \to \mathbf{B},\ \mathbf{B} \to -\mathbf{J}^{m})$$

$$\mathbf{J}^{m} = -(2e\hbar/mc)\mathbf{A},\quad \mathbf{B} = \mathrm{curl}\mathbf{A} \equiv \mathrm{curl}(\zeta \mathbf{J}^{m}),$$

$$\zeta = (mc/2|e|\hbar)$$

$$\mathrm{Det}\|F_{\mu\nu} - \lambda\eta_{\mu\nu}\| \equiv \lambda^4 - (R_{\mu\nu\rho\sigma}F^{\mu\rho}F^{\nu\sigma})\lambda^2 + (\in_{\mu\nu\rho\sigma}F^{\mu\rho}F^{\nu\sigma})^2 = 0,$$

$$\mathrm{Det}\|G_{\mu\nu} - \lambda\eta_{\mu\nu}\| \equiv \lambda^4 - (R_{\mu\nu\rho\sigma}G^{\mu\rho}G^{\nu\sigma})\lambda^2 + (\in_{\mu\nu\rho\sigma}G^{\mu\rho}G^{\nu\sigma})^2 = 0,$$

$$\|\eta_{\mu\nu}\| \equiv diag(+1,-1,-1,-1), R_{\mu\nu\rho\sigma} \equiv (\eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\rho}\eta_{\sigma\nu}),$$

$$R_{\mu\nu\rho\sigma}F^{\mu\rho}F^{\nu\sigma} - 2 \in_{\mu\nu\rho\sigma} F^{\mu\rho}F^{\nu\sigma} = \begin{cases} \mathbf{E}^2 \\ \mathbf{B}^2 \end{cases} \Rightarrow \begin{cases} \mathbf{E}^2 - \mathbf{B}^2 \pm 2\mathbf{B}.\mathbf{E} = 0 \\ \mathbf{B}^2 - \mathbf{E}^2 \pm 2\mathbf{E}.\mathbf{B} = 0 \end{cases}$$

$$R_{\mu\nu\rho\sigma}G^{\mu\rho}G^{\nu\sigma} - 2 \in_{\mu\nu\rho\sigma} G^{\mu\rho}G^{\nu\sigma} = \begin{cases} \mathbf{J}^{m2} \\ B^2 \end{cases} \Rightarrow \begin{cases} \mathbf{J}^{m2} - \mathbf{B}^2 \pm 2\mathbf{B}.\mathbf{J}^{m} = 0 \\ \mathbf{B}^2 - \mathbf{J}^{m2} \pm 2\mathbf{J}^{m}.\mathbf{B} = 0 \end{cases}$$

Note that the **current loop** J^m, involves **e/m**, and this insures the **Meissner effect** ($J^m.B \neq 0$)

Shown below is the hyperlink model of the H-atom characterized in ref. [14], by Alexander Animalu, Samuel Edeagu, Godfrey Akpojotor & Erik Trell, Electromagnetism of Atomic Structural Constitution in Deformable Real R3xSO(3) Configuration Space, Afr. J. Phys. 11, 2 (2018).

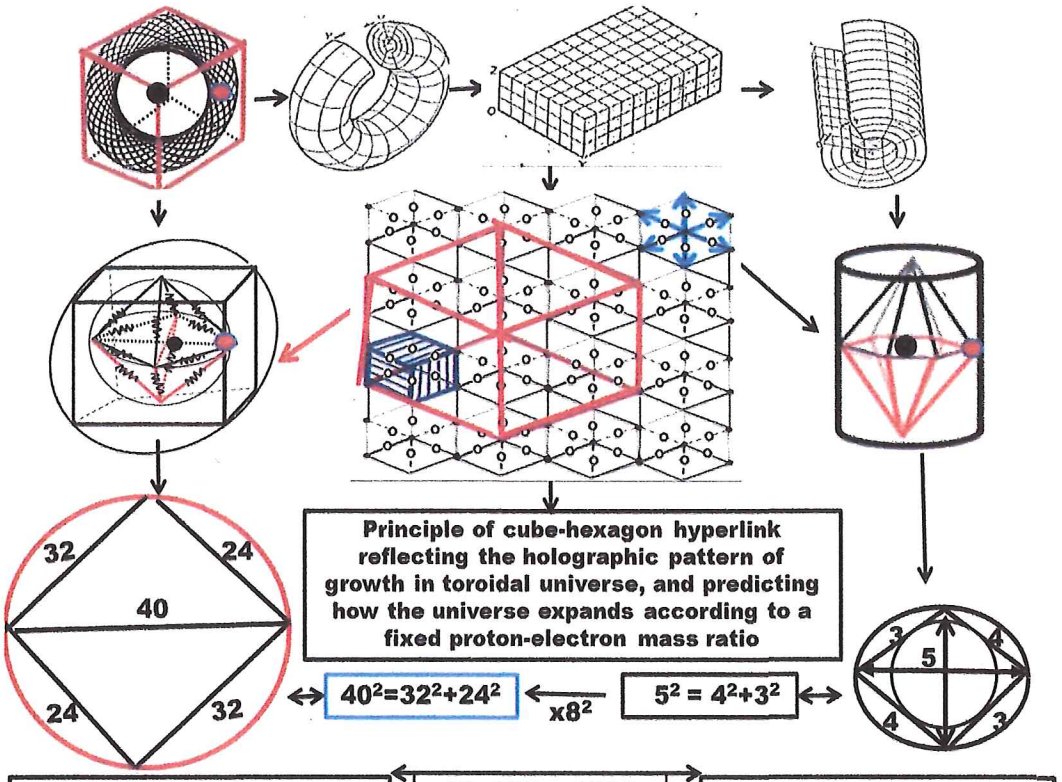


Fig. : Model of H-atom(top-left)

. From the left, Profs. R. M. Santilli, A. O. E. Animalu and D. D. Shillady

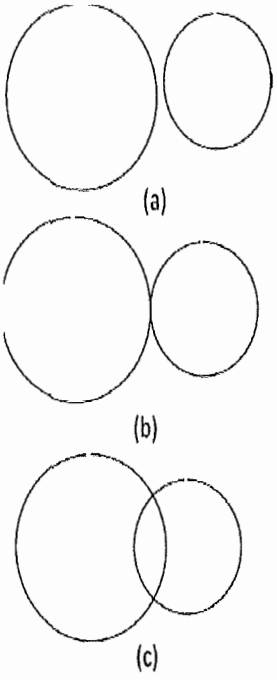
The new forces of interest in this paper were first identified in physics and verified by R.M. Santilli with the representation of the synthesis of mesons (in Section 5 of his 1978 Harvard paper) [2]. Following that, in collaboration with Alex Animalu (as former University of Cambridge research physicist), the new mathematics was applied to the representation of the *attractive force* between the identical electrons of the Cooper pair in superconductivity [6]. In view of encouraging results in superconductivity, systematic studies lead to the first and only known 'attractive force' between valence electrons pairs in *molecular structures* nowadays known as the *isoelectronium.* In collaboration with the chemist Don D. Shillady of Virginia Commonwealth University, it was proved that the 'completion' of quantum chemical models into the form admitting an explicitly attractive force between valence electron pairs achieved an exact representation of experimental data of the hydrogen molecule, and of the water molecule. The progressive conceptualization is summarized in Fig.1 and elaborated further in Fig.2(a)-2(d) below and the correspondence in quasi-crystalline array of water molecules, between current loops and magnetic poles as shown in Fig. 3.

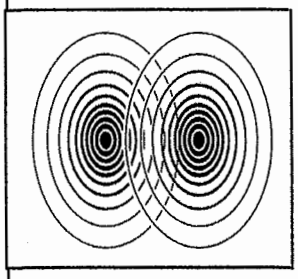
Fig 1

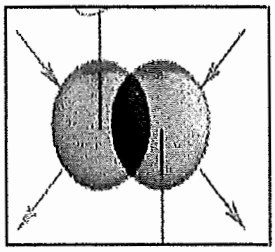
(a) Schematic representation of the separation between the Compton wave packets of e^- and p in normal H-atom

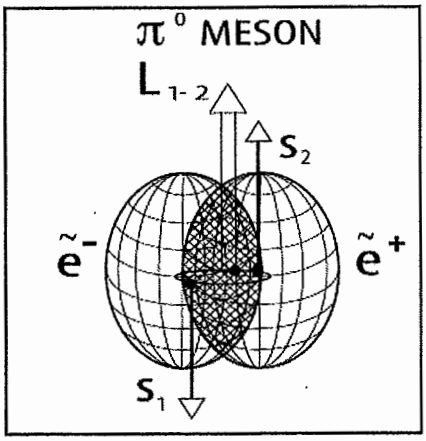
(b) Contact of the Compton wave packets of e^- and p at the threshold for pe^- fusion into a "neutral doublet" pe^-

(c) Overlap of the Compton wave packets of e^- and p under fusion.









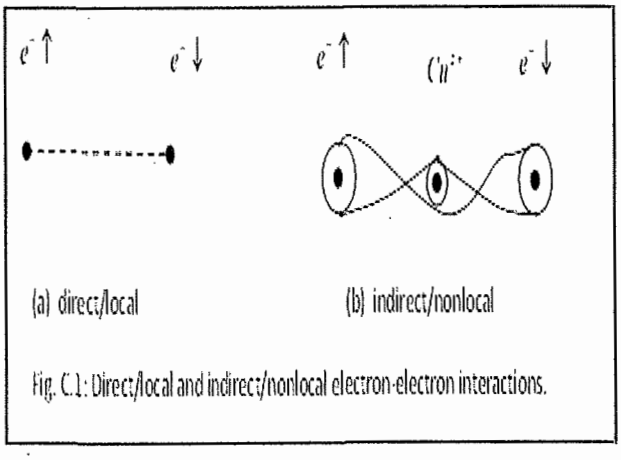


Fig. C.1: Direct/local and indirect/nonlocal electron-electron interactions.

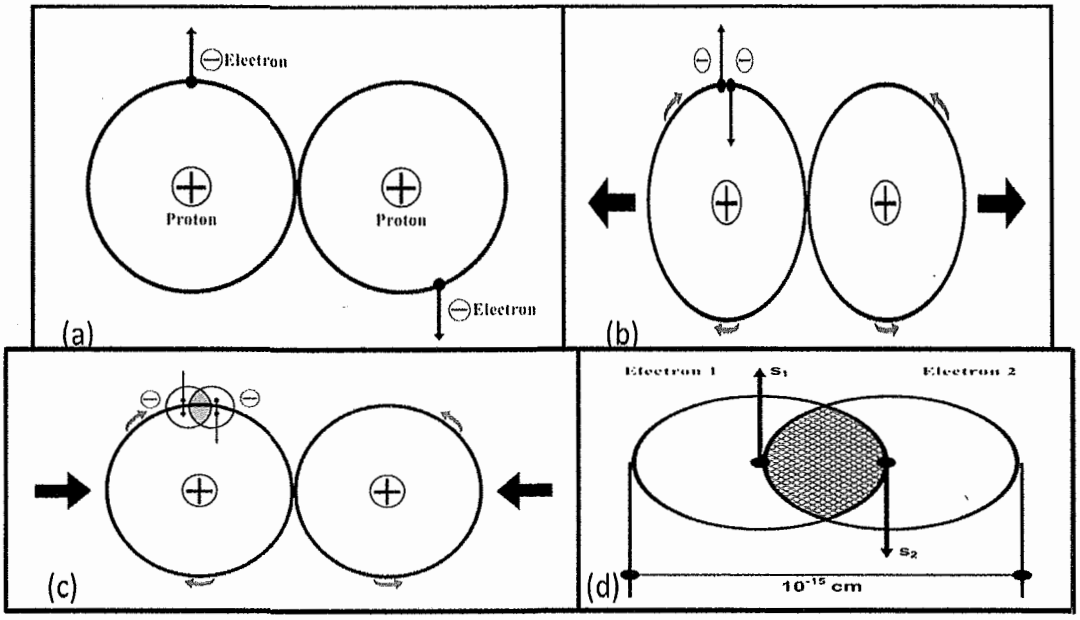


Fig.2(a): A conceptual rendering of the hydrogen molecule at absolute zero degree temperature without the bond of valence electrons showing *no attractive or repulsive force between the two atoms* between the two atoms.

2(b): A conceptual rendering of the hydrogen molecule according to quantum chemistry showing the repulsive (rather than a bond) between two point-like valence electrons due to their equal charge.

2(c) A conceptual rendering of the *hydrogen molecule according to the completion of quantum chemistry into hadronic chemistry* illustrating the first and only known attractive force between valence electrons caused by *non-potential interactions due to deep entanglement of extended wavepackets.*

2(d): A conceptual rendering of the structure of the isoelectronium denoting two valence electrons with *antiparallel spins* under deep penetration *of their wavepackets,* as occurring in reality.

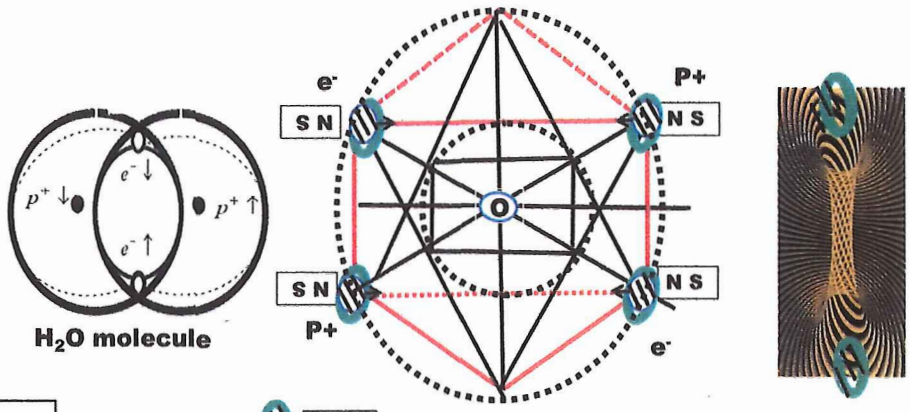


Fig. 3: Current loop → N S ← Equivalent Magnetic poles

2. BROADENING OF DUAL DICHOTOMY TO TRICHOTOMY

Dual dichotomy is a description of how to construct a Lie-admissible broadening of the Lie-isotopic scattering theory of HM for time-irreversible spin- 1/2 particle events in terms of the "lifting" (i.e., "genetopy") of a non-diagonal underlying metric of (ct,r)-space defined by three discrete symmetry transformations of charge conjugation (C), time-reversal (T) and space-time reciprocity analogous to the conventional duality (D) of electric and magnetic fields in electromagnetism. Geometrically, the genotopy characterizes a topological-deformation(ε) of a sphere in r-space into a torus in two ways defined in terms of the underlying non-diagonal metrics by

$$\eta \equiv \begin{pmatrix} 1 & -\varepsilon \\ \varepsilon & 1 \end{pmatrix} \rightarrow \begin{cases} \begin{pmatrix} -1 & \varepsilon \\ \varepsilon & 1 \end{pmatrix} \equiv {}^{<}\eta \\ \begin{pmatrix} -\varepsilon & 1 \\ 1 & \varepsilon \end{pmatrix} \equiv \eta^{>} \end{cases}$$

However, from the Lie-admissible formulation of "extended" relativity principle in terms of Dirac γ_μ matrices we know that, physically, genotopy transforms a vector into an axial-vector. Also T-symmetry is not purely classical but has an operator counterpart in conventional quantum mechanics(QM), its non-unitary character demands a further generalization of QM over real and complex numbers to the Lie-admissible covering HM over genonumbers, genofields, … and for consistency, a generalization here called "Trichotomy" of conventional scattering theory of QM to genoscattering theory of HM, and conventional model of two molecules

($2H_2O$) "water current" as reactant "hydroxil ion" consisting of oxonium(H_3O^+) cloud and hydroxide(OH^-) biogas to be characterized in SO(10) linguistics, for correlated definitions of "hadronc conductivity" & "superconductivity"

3. CORRELATED DEFINITIONS OF HADRONIC CONDUCTIVITY AND SUPERCONDUCTIVITY

Specifically, SU(3) algebraic geometric trichotomy emerges via a correspondence principle,

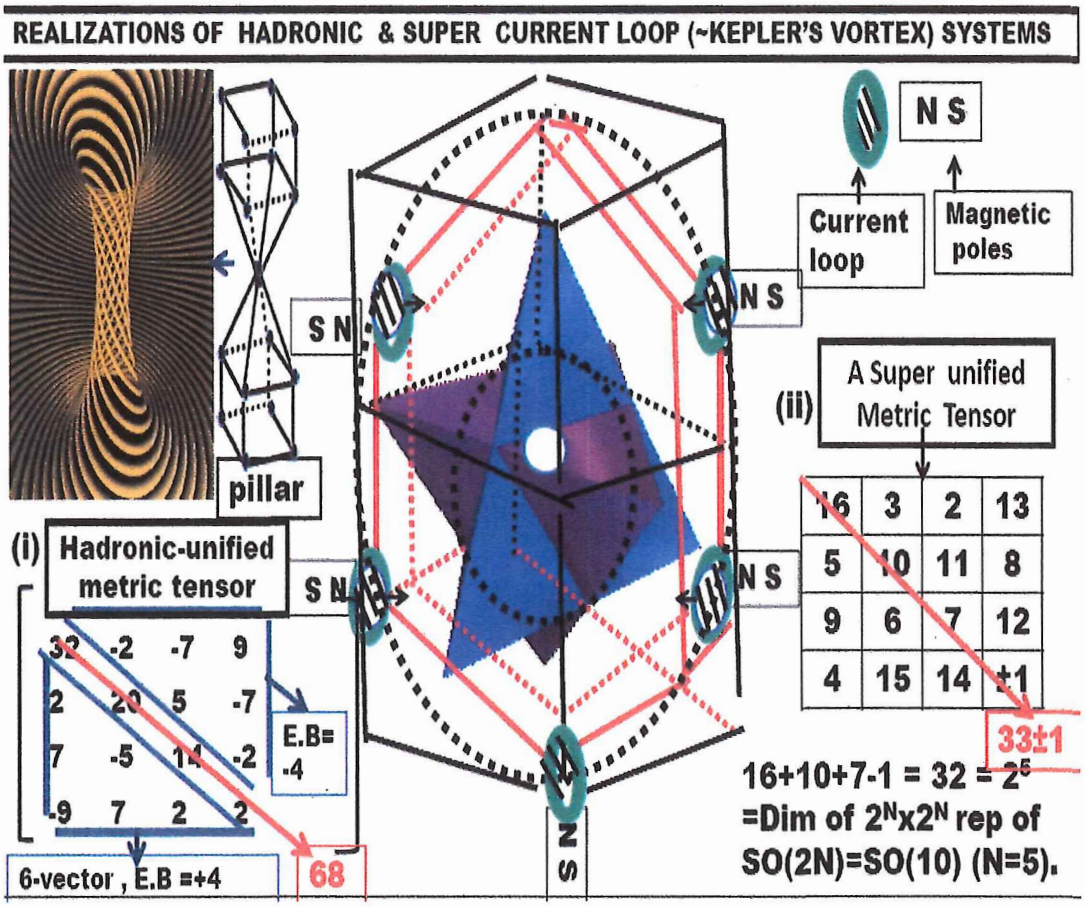
points↔particles, lines↔fields and planes↔currents,

between (points, line and planes of 3-dimensional projective geometry) and (particles, field and currents of Maxwell-type gauge-field theory) from the Lie-admissible product generated by transformation of a point-sphere into a torus immersed in a cube-hexagon genospace.

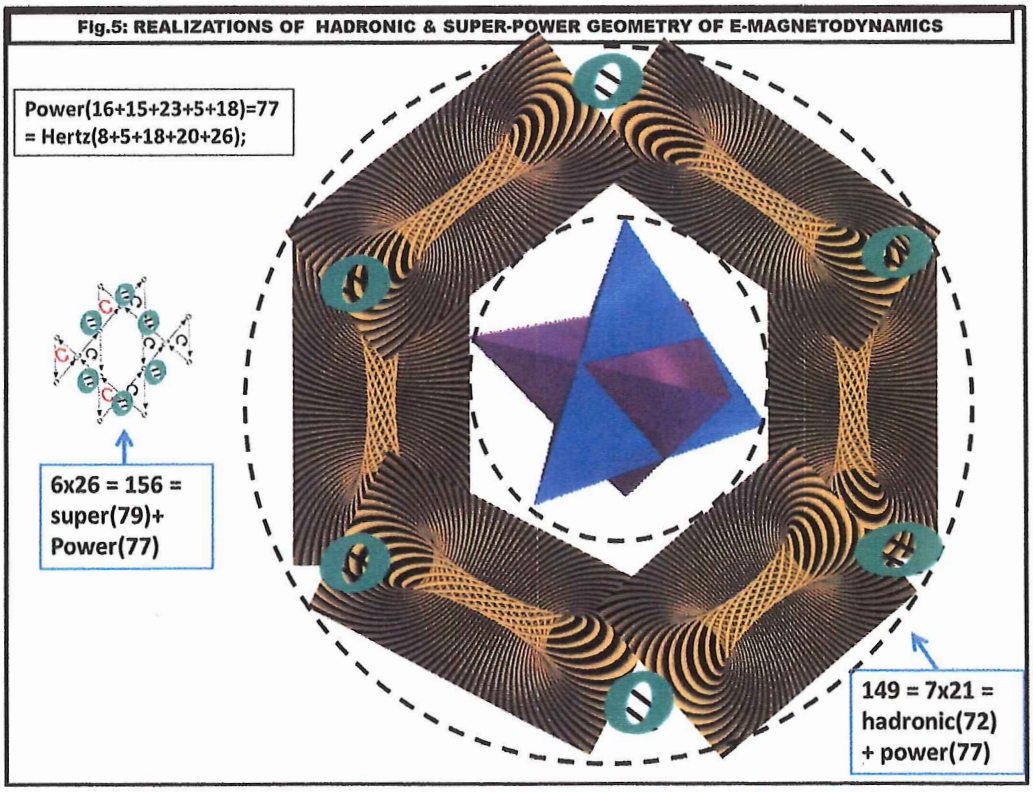
Analogue	Digital Value	(*) Remark
Conductivity	Conductivity(3+15+14+4+21+3+20+9+22+9+20+25=163)	Process $2H_2O$ $H_3O^+ + OH^-$ yields oxonium + hydride ; and e-Hydroxil Is divalent. See, [14] Ref. O.Y. Ababio, New School Chemistry Certificate Science Series p.123, 131 & 294
Water Current	water(23+1+20+5+18=67)+current(3+21+18+18+5+14+20=99)=67+99 =166	
*Oxonium Cloud	Oxonium(15+24+15+14+9+21+13=111)+ cloud(3+12+15+21+4=55) = 111+55= 166	
*A hydride Biogas	A(=1) hydroxide(8+25+ 4+18 +15+24+9+4+5=112) +biogas(2+9+15+7+1 +19=53) =1+112+53 = 166	
* e-Hydroxil	(5+8+25+4+18+15+24+9+12=120)	
Hadronic iso e-hydroxil = Hadronic conductivity	hadronic(8+1+4+18+15+14+9+3=72) iso(9+19+15=43) e-hydroxil(5+8+25+4+18+15+24+9+12=120)= 72+43+120=235=72+163 = hadronic(=72)+Conductivity(=163)	
Super Iso e-hydroxil = =Super conductivity	super(19+21+16+5+18=79)+ Iso(9+19+15=43) +e-hydroxil(5+8+25+4+18+15+24+9+12=120) = 79+43+120=242=79+166 =super(=79)+Conducivity(=163)	

Thus, in SO(10) linguistic geometry , correlated definitions of Hadronic conductivity and Superconductivity emerge in the analogue/digital form from the entity "e-hydroxil" as the reactant ion of two water molecules ($2H_2O$).

Realization of current loops & associate hadronic-unified metric tensor as shown in Fig.4(i) is characterized by electric (E=2,-7,9) & magnetic (B=2,-7,5) 6-vector components, such that E.B=±4 animated as "pillar of water" ; whereas super current loop & associated super-unified metric tensor as shown in Fig.4(ii) is characterized by an array of magnetic poles & pivoted slab of soft iron in e-magnetodynamics machine(Animalu et al 2013) animated as array of magnetic poles with pivoted slab replaced by Kepler's 6-star vortex. The number of (points+lines+planes)are: Pillar(68), Cube (26), Vortex(7x6=42), Hexagon(38) & 34+38=72(="hadronic") system; and 42+38=80(="A Super") system.



Moreover, by observing that in SO(10) linguistics current(3+21+18+18+5+14+20=99) + Loop(12+15+15+16 =58) = 157= 1+ 156 where 156=6x26=79+77= super(=79)+power(16+15+23+5+18=77), and "Hadronic"(=72)+ power(=77)=149= 7x21, we have the gathering of geometric relations and power relations shown below in Fig.5.



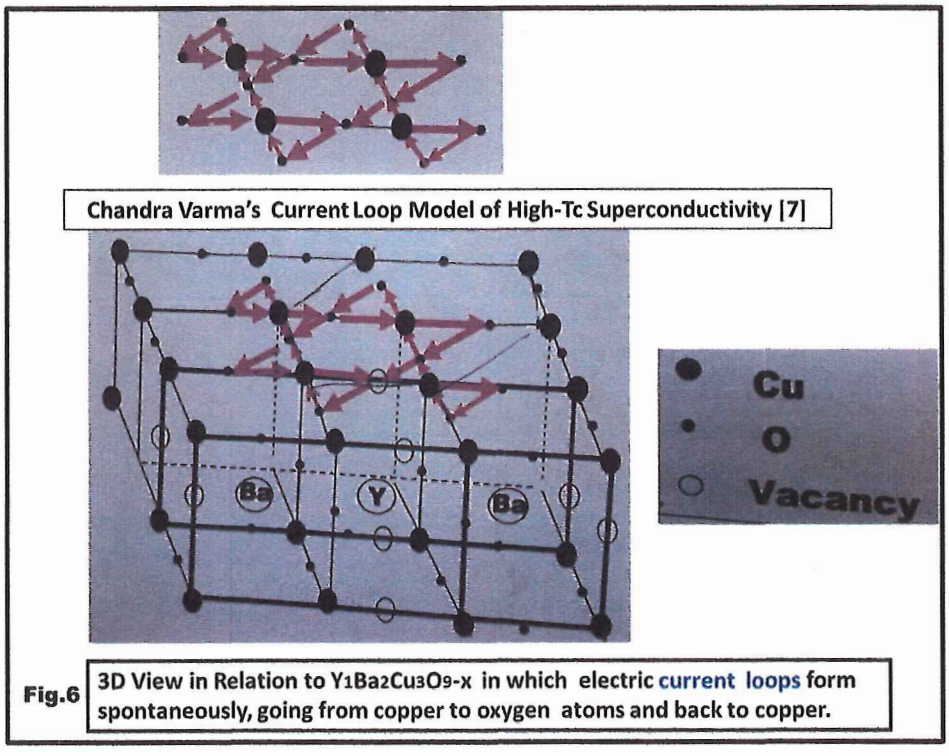


Fig.6 3D View in Relation to $Y_1Ba_2Cu_3O_{9-x}$ in which electric current loops form spontaneously, going from copper to oxygen atoms and back to copper.

4. USE OF EPR ARGUMENT FOR RELATING HADRONIC AND SUPER CONDUCTIVITY

In A PubRelco Interview of R.M. Santilli with Scientific and Industrial implications New York, N.Y., April 15, 2019 In response to the question: "Prof. Santilli, could you please **review in a language** accessible to the general audience **Einstein's 1935 historical prediction that quantum mechanics and, therefore, quantum chemistry are incomplete theories",** the following answer was given: "Einstein did not accept the **uncertainty of quantum mechanics,** including the impossibility to identify the position of a particle with classical precision. For that reason, he made his famous quote **"God does not play dice with the universe."**

Einstein **accepted quantum mechanics for atomic structures, but believed that quantum mechanics is an "incomplete theory,"** in the sense that it could be broadened into such a form to **recover classical determinism** at least under special conditions. The same argument also applies to **quantum chemistry**". It admits interior entanglement/Lie-admissible penetration which is characterized in Refs.[1]-[3] by progressive generalization of the **Lie-algebraic product** of quantum mechanics (and hence quantum chemistry) in **"hadronic mechanics"** as follows:

$$AB-BA\rightarrow\begin{cases} APB-BP^{\dagger}A,(P^{\dagger}=P=I)\rightarrow \text{Conventional Lie algebraic product} \\ APB-BP^{\dagger}A,(P^{\dagger}=P\neq I)\rightarrow \text{Lie isotopic algebraic product} \\ APB-BP^{\dagger}A,(P^{\dagger}\neq P\neq I)\rightarrow \text{Lie admissible algebraic product,} \end{cases}$$

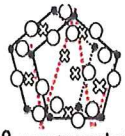
This progressive Lie-admissible algebraic structure has been realized since (2011) in **geometric terms as a deformation of a point sphere into a torus** characterized in the framework of **"Non-unitary scattering theory of hadronic mechanics"** by R.M. Santilli and A.O.E. Animalu Refs.[4-8]

In view of the linguistic challenges of hadronic mechanics, we have done linguistic geometric elaboration of the EPR argument in analogue/digital SO(10) characterization as shown below for iso-spin trichotomy and iso-spin dual dichotomy, and for hadronic and super conductivity exemplified by 2 molecules ($2H_2O$) "water current" in the reactant, "hydroxil"↔ oxonium(H_3O^+) + hydroxide(OH^-),
states as "oxonium cloud" & "hydroxide biogas".

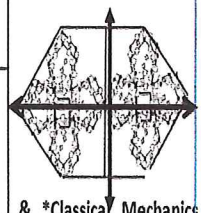
LINGUISTICS GEOMETRIC ELABORATION OF THE EPR ARGUMENT FOR ANALOGUE/DIGITAL SO(10) CHARACTERIZATION OF ISO-SPIN HADRONIC ENERGY , ISO-SPIN DUAL DICHOTOMY & ISO-SPIN TRICHOTOMY

Analogue	Digital (GENO-ASCII Code, A ~ 1,B~2,C~3,...,Z~26, so that HADRON ~ H+A+D+R+O+N=8+1+4+18+15+14=60; & (A,B,C,...Z)~(a,b,c,...,z), such that in SO(10),1+2+3+....+36=666.
*Life; Cell; Bit	12+9+6+5=**32 = 2^5** ; Cell= 3+5+12+12=**32** ; Bit= 2+9+20=**31**
***Classical Mechanics**	(**3+12+1+19+19+9+3+1+12**)+(13+5+3+8+1+14+9+3+19)=**79+75**=155
A Quantum Mechanics **Particle Physics** A Nuclear Dodecahedron	1+(17+21+1+14+20+21+13=**107**)+(13+5+3+8+1+14+9+3+19=75) =1+107+75 = 183; (16+1+18+20+9+3+12+5=84)+(16+8+25+19+9+2+19=99) = 84+99=**183.** 1+(14+21+3+12+5+1+18=84)+(4+15+4+5+3+1+8+5+4+18+15+14=**98**)=**183**
Wave Particle Duality **Quantum Chemistry** **A Dichotomy Exterior**	(23+1+22+5=51)+(16+1+18+20+9+3+12+5=84)+(4+21+1+12+9+20+25=92) =**227** (17+21+1+14+20+21+13=**107**)+(3+8+5+13+9+19+20+18+25=**120**)=**227** (1+4+9+3+8+15+20+15+13+25=113)+(5+24+20+5+18+9+15+18=**114**)=**227**
Quantum Mechanics **Is not Complete** Wave-Particle Duality **Theory**	(17+21+1+14+20+21+13=**107**)+(13+5+3+8+1+14+9+3+19=**75**)+ (9+19)=28)+(14+15+20=49)+(3+15+13+16+12+5+20+5=**89**)+ (23+1+22+5=51)+(16+1+18+20+9+3+12+5=84)+(4+21+1+12+9+20+25= 92) + (20+8+5+15+18+25=91)= **107+75+28+49+89+51+84+92+91 = 666**
God Does Not Play A Dice with the Universe **Entanglement Interior**	(7+15+4=26)+(4+15+5+19=43)+(14+15+20=49)+(16+12+1+25=54)+ (1+4+9+3+5=22)+(23+9+20+8=60)+(20+8+5=33)+(21+14+9+22+5+18+9+5=113)+ (5+14+20+1+14+7+14+5+13+5+14+20=148)+(9+14+20+5+18+9+15+1=108) = **26+43+49+54+22+60+33+113+148+108 = 666**
***A Complete Time Determinism**	(1+3+15+13+16+12+5+20+5=80)+(**20+9+13+5=47**)+ **4+5+20+5+18+13+9+14+9+19+13=129) = 80+47+129=256**
***Iso-spin Trichotomy ;** Pascal Triangle(N=8)line	**(9+19+15+19+16+9+14=101)+(20+18+9+3+8+15+20+15+13+25=145) =** **101+145=256 ; 1+8+28+56+70+56+28+8+1 = 256**
Iso- Spin Dual **Dichotomy**	(9+19+15+19+16+9+14=101)+(4+21+1+12=38)+ (4+9+3+8+15+20+15+13+25=112) = **101+38+112=251**
Iso-spin Hadronic Energy	101+(8+1+4+18+15+14+9+3=72)+(5+14+5+18+7+25=74)=**101+72+74=257**

(*) Remarks :
In SO(10) group representation of ***Life ("cell") by**
32 = 10 points + 15 lines + 7 planes.



& representation of *"A complete time determinism" (**$256=2^8$**) cells by



& *Classical Mechanics is given by **5 Bits (5x31=155)** Forming Platonic **Icosahedron dual** of a dodecahedron.

Fig. 7 : Characterization of Pascal Triangle for Water Molecule

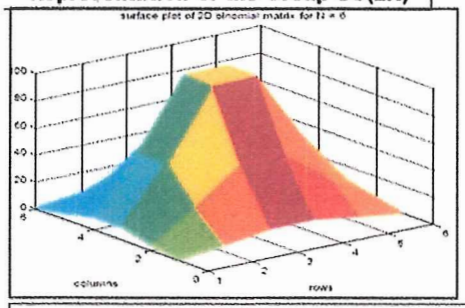
1	$N=0,\ 2^0=1$
1 1	$N=1,\ 2^1=2$
1 2 1	$N=2,\ 2^2=4$
1 3 3 1	$N=3,\ 2^3=8$
1 4 6 4 1	$N=3,\ 2^4=16$
1 5 10 10 5 1 ⟶	$N=5,\ 2^5=32$
1 6 15 20 15 6 1	$N=6,\ 2^6=64$
1 7 21 35 35 21 7 1	$N=7,\ 2^7=128$
1 8 28 56 70 56 28 8 1	$N=8,\ 2^8=256$

Pascal Triangle for the $2^N x 2^N$ matrix

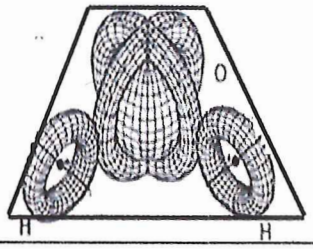
[1	5	10	10	5	1	→ 16+16
5	25	50	50	25	5	→ 80+80
10	50	100	100	50	10	→151+151
10	50	100	100	50	10	→151+151
5	25	50	50	25	5	→ 80+80
1	5	10	10	5	1]	→ 16+16

6x6 Pascal Triangle Matrix

Representation of the Group SO(2N)



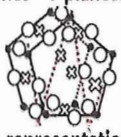
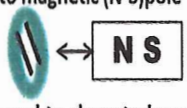
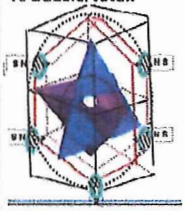
Surface plot of 6x6 Pascal Triangle matrix



H-O-H Plane of Water Molecule

Water molecules (H_2O) have essentially null electromagnetic features, yet in SO(10) linguistic geometry, they bond as ice(9+3+5=17) at 1+16=17 (o Fahrenheit) and form Water(23+1+20+5+18=67) as "Pillar"(16+9+12+12+1+18=68) at 1+67=68 before break-up at boiling point 100. Accordingly, Pascal triangle provides a key to Santilli's H-O-H model of H_2O .

LINGUISTICS GEOMETRIC ELABORATION OF THE EPR ARGUMENT FOR ANALOGUE/DIGITAL SO(10) CHARACTERIZATION OF PARTICLE-WAVE AS ELECTRONIUM & ISO-ELECRONIUM AS CURRENT LOOP IN HIGH-Tc SUPERCONDUCTIVITY

	Digital (GENO-ASCII Code, A ~ 1,B~2,C~3,...,Z~26, so that HADRON ~ H+A+D+R+O+N=8+1+4+18+15+14=60; & (A,B,C,...Z)~(a,b,c,...,z), such that in SO(10),1+2+3+....+36=666.	(*) Remarks
Analogue	**Digital**	In SO(10) group representation of *Life ("cell") by 32 = 10 points + 15 lines + 7 planes.  a representation of iso-electronium is given by 178 = "loop of current" equivalent to magnetic (N-S)pole  used to characterize a supercurrent in high-Tc quasicrystal. 
Wave Particle	(23+1+22+5=51); (16+1+18+20+9+3+12+5=84); 51+84=**135**	
Electronium	5+12+5+3+20+18+15+14+9+21+13= 135	
*Iso-electronium	(9+19+15=**43**)+(5+12+5+3+20+18+15+14+9+21+13=**135**)= **178**	
*Loop of current	(12+15+15+16=**58**)+(15+6=**21**)+(3+21+18+18+5+14+20=**99**)=**178**	
Supercurrent	(**19+21+16+5+18=79**)+(3+21+18+18+5+14+20=**99**)= **178**	
Superconductivity	(19+21+16+5+18=**79**)+(3+15+14+4+21+3+20+9+22+9+20+25=**163**)= **242**	
IDENTIFICATION OF A SUPER ISO-ELECTRONIA WITH HADRONIC CONDUCTIVITY		
Hadronic-current	(8+1+4+18+15+14+9+3=**72**)+(3+21+18+18+5+14+20=**99**)= **171**	
Electronia	5+12+5+3+20+18+15+14+9+1=**102**	
Vortex	**22+15+18+20+ 5+24 = 102**	
*Iso-electronia	(9+19+15=**43**)+(5+12+5+3+20+18+15+14+9+1=**102**) = **155**	
A Super Iso-electronia	1+(19+21+16+5+18=79)+(9+19+15=**43**)+(5+12+5+3+20 +18+15+14+9+1=**102**) = 235	
A Water current Pillar	1+(23+1+20+5+18=67)+(3+21+18+18+5+14+20=99)+(16+9+12+12+1+18=68) = 235	
Hadronic conductivity	(8+1+4+18+15+14+9+3=**72**)+(3+15+14+4+21+3+20+9+22+9+20+25=**163**)=235	

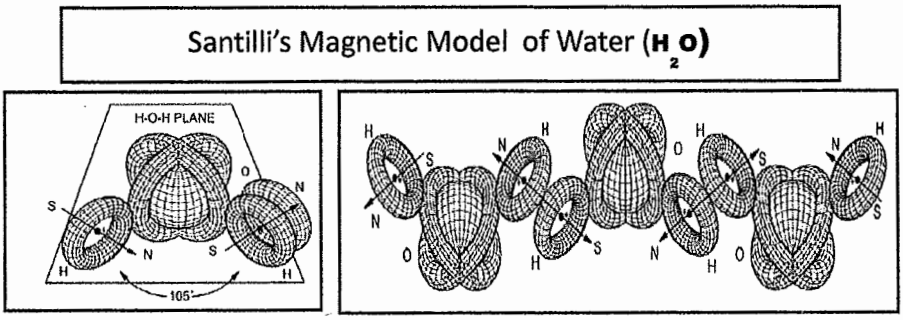


Fig.8 (left). A conceptual rendering of anomalous species (H_3O^+), as a possible quantitative model of the H-bonds in the liquid state of water. Note that the water molecule is represented as it occurs in nature, namely, with polarization of the orbitals perpendicular to the H – O – H plane, thus possessing a molecular structure particularly suited for the magnecular bond H × H ; Figure(right). One of the possible magnecular bonds of H-atoms in the liquid state of water according to (Ref. [1]). The second expected magnecular H-bond as in the left figure is that with the O-atom, resulting in the typical lattice structure of the liquid state.

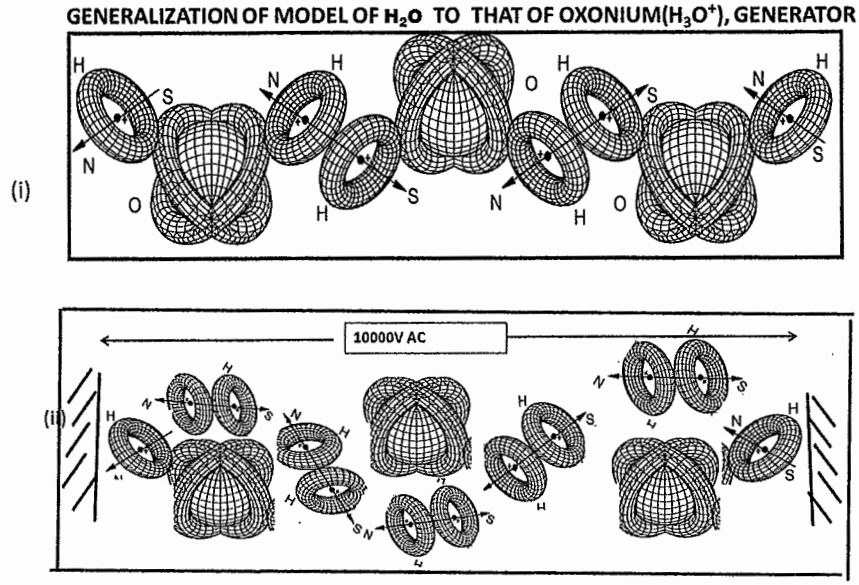


Fig. 9(i) Santilli's model of liquid state of water(H_2O) showing TWO "isoelectronia", with parallel orbits & (ii) Generalization as power generator showing FIVE "isoelectronia" with synchronized magnetic NS poles in the Oxonium (H_3O^+) state of ($2H_2O$) water molecules.

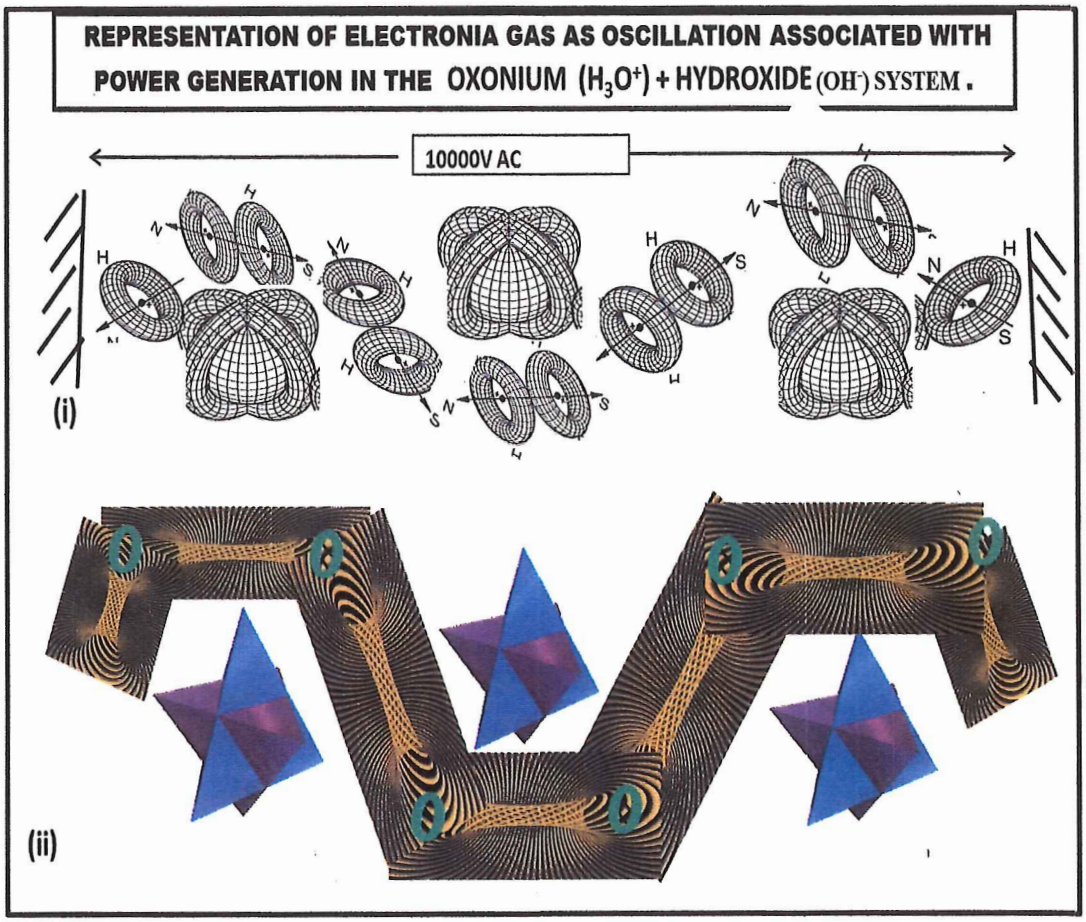
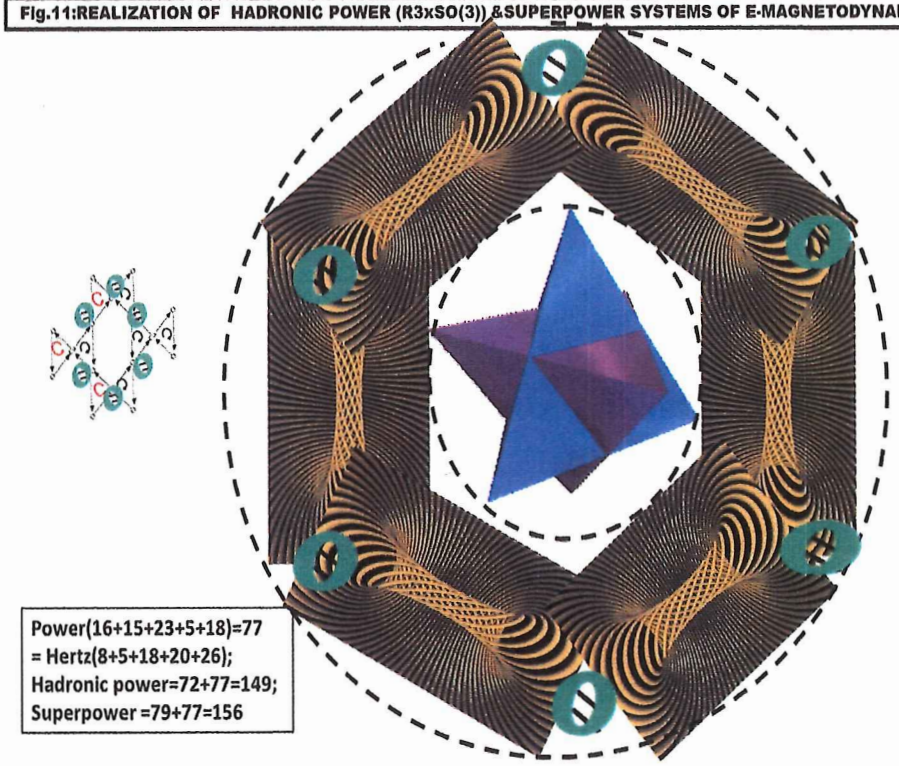

REPRESENTATION OF ELECTRONIA GAS AS OSCILLATION ASSOCIATED WITH POWER GENERATION IN THE OXONIUM (H_3O^+) + HYDROXIDE (OH^-) SYSTEM .
10000V AC
(i)
(ii)

Fig.10: (i) e-hydroxyl generator showing FIVE "isoelectronia" (P^+e^--pair of H) with synchronized magnetic NS poles in Oxonium (H_3O^+), . (ii) representation of electronia gas as oscillation of connecting rods of current loops alternating with vortex motions of the oxygen in the ionized($2H_2O$) ↔ Oxonium (H_3O^+) + hydroxide(OH^-) states with oscillation angle of 105^0.

Fig.11:REALIZATION OF HADRONIC POWER (R3xSO(3)) &SUPERPOWER SYSTEMS OF E-MAGNETODYNAMICS



5. DISCUSSION AND CONCLUSION

In this paper we have, on the strength of the EPR Argument, developed in the framework of hadronic mechanics, both a theoretical physics (e-magnetodyamics framework of current loops) and linguistic (analogue/digital) SO(10) realizations of a principle of trichotomy between the three objects (points, lines, planes) of projective geometry of space-time and the three elements (particles, fields, currents) of gauge principles of classical and quantum mechanics . As specific example, we have done a progressive characterization of the hydrogen atom(H), water molecule (H_2O) and the ionized state of $2H_2O$ Oxonium (H_3O^+) + hydroxide(OH^-) with respect to a new form of high-temperature superconductivity ("super iso e-hydroxil") as hadronic conductivity ("hadronic iso e-hydroxil") and in terms of pairing mechanism; we have also shown that "a super iso-electronia" is identifiable with "hadronic conductivity". The relevance of hadronic(=72) power (=77) = 149 can be inferred from the fact that it is cognate with "environment"(5+14+22+9+18+15+14+13+5+14+20=149), which is a topical issue in this era of global warming and challenges of

COVID-19(3+15+22+9+4=72) pandemic. But most important is the implication of trichotomy of "time-relativity" in real R3xSO(3), hereunder called T-symmetry which is discretely positive but non-linear and irreversible according to *"Energy=Love x Square of Speed of Light(c) "* unified theory of energy generation in everything in Einstein's Secret 1938 Letter to his daughter(Lieseri) 1 year before the 2nd World War (1939) which was released in 1980 as reported by Brandon Bays(www.thejourney.com)[15] as "God is Love and Love is God" ; "Love is the quintessence of universal energy generation ..."; etc. The significance of T-symmetry is correlated hereunder with the trichotomy of world war dates (DAY/MONTH/YEAR)~(ab/cd/efgh) in analogue/digital SO(10) linguistics of life(12+9+6+5=32), language(12+1+14+7+21+1+7+5=68), and power(16+15+23+5+18=77) = 47+30 = time(20+9+13+5=47)axe(1+24+5=30) consistent with Einstein's secret letter *vis-à-vis* (40+1+40=81) word of James Cameron(TikTok@nickygumbel) in the Movie "Titanic(20+9+20+1+14+9+3=76)" built with new technology to be "unsinkable" but sank on 15/04/1912 with loss of 1513(=666+81+666) people(16).

Trichotomy of "T-symmetry" in real R3xSO(3) and its correlation in analogue/digital SO(10) linguistics of war dates(DAY/MONTH/YEAR)~ab/cd/efgh, defined by (T1,T2,T3):T1= a+b+c+d+e+f+g+h;T2=ab+cd+ef+gh,T3=ab+c+d+e+f+g+h characterize life, language & power & Einstein's 1938 secret letter; and digitally "hadronic conductivity" ~ "God is Love and Love is God" in SO(10) as follows:

Date of 1st World War, 28 /07/1914 gives T1=2+8+0+7+1+9+1+4=32; T2=28+7+19+14 = 68, T1+T2=68+32=100 T3=28+0+7+1+9+1+4=50;T1+T3=68+50 = 118	The 1st world war gives, T1=life(12+9+6+5=32) T2=language(12+1+14+7+21+1+7+5=68) while T3 =50 = t-axe(20+1+24+5=50) is equivalent to time(20+9+13+5=47)-axe(1+24+5=30) =47+30= 77 = power(16+15+23+5+18=77); so that T1+T3 =50+68=118 = 44+74=anti(1+14+20+9=44) nuclear(14+21+3+ 12+5+ 1+18=74). Hence the 1938 secret letter one year before the 2nd World War limited initially to preservation of life and language but with discovery of nuclear fission, it ended with geno(=41)power(=77) =118,(cf, 77= Titanic(20+9+20+1+14+9+3=76)+1.
Date of 2nd World War, 1/09/1939 gives T1=1+0+9+1+9+3+9=32; T2=1+9+19+39 = 68; T1+T2=68+32=100 T3=1+0+9+1+9+3+9=32; T1+T3=68+32=100	
Date of sinking of "Titanic" 15/04/1912 gives T1=15+4+1+9+1+2=32; T2=15+4+19+12=50; T1+T2=32+50=82 T3=1+5+4+1+9+1+2=23; T1+T3=32+23=55	

Hadronic(8+1+4+18+15+14+9+3=72) conductivity (3+15+14+4+21+3+20+9+22+9+ 20+25=163) = 235 = (26+28+54)+19+(54+28+26) = God(7+15+4=26) is(9+19=28) Love(12+15+22+5=54) and(1+14+4=19)Love(12+15+22+5=54) is(9+19=28) God(7+15+4=26), as stated in secret letter.

Love(=54) is(=28) the(20=8+15=33) quintessence (17+21+9+14+20+5+19+19+5 +14+3+5= 151) of(15+6=21) universal (21+14+9+22+ 5+18+19+1+12= 121)+ energy (5+14+5+18+7+25 =74) generation(7+5 +14+5+18 +1+20+9+15+14=108) pyramid(16+25+18 +1+13+9+4=86) = 54+28+33+151+21+121+ 74+ 108+86 = 666 = 1+2+3+...+36 (for closure in SO(10) linguistics). & 666+81+666=1513 is the number drowned according to the 81= "word(23+15+18+4=60)of(=21)" Cameron (Tik Tok@nickygumbel) in the Movie , "Titanic", built with new technology to be "unsinkable", but sank by cracks(3+18+1+3+11+19 = 55) on 15/4/1912..

ACKNOWLEDGEMENTS

I wish to thank Professor Ruggero M. Santilli for his interest and guidance in the direction of this research project for decades (since Jun 23, 2021) and for the material support of The Institute for Basic Research; and Professor Charles Igwe, Vice-Chancellor of the University of Nigeria, Nsukka for his support and interest in promotion of basic research at various university institutional levels of excellence.

6. REFERENCES

[1] P.A.M. Dirac 1971 Proc. Roy. Soc. 322A, 435 (1971).

[2] R.M. Santilli 1978 *Hadronic J.* 1, 574

[3] A. Einstein, B. Podolsky and N. Rosen (EPR), (1935) "Can quantum-mechanical description of physical reality be considered complete?" Phys. Rev., Vol. 47, p. 777 (1935), www.eprdebates.org/docs/epr-argument.pdf
Nine minutes video on EPR , verifications,
http://www.world-lecture-series.org/legacy-of-einstein-for-new-clean-energies
Tutoring Lecture I: Isomathematics http://www.world-lecture-series.org/santilli-tutoring-i
Tutoring Lecture II: Verifications of the EPR argument
http://www.world-lecture-series.org/santilli-tutoring-ii
Primary references on recent verifications of the EPR argument
http://www.santilli-foundation.org/docs/references-epr-verifications.pdf

[4] E. Recami: Multi-verses, *Micro-universes and Elementary Particles (Hadrons)* arXiv:hep-th/0505190 v1 21 May 20051978

[5] R. M. Santilli & A.O.E. Animalu *2011 Non-unitary Lie-isotopic & Lie-admissible scattering theory of HM* (2 011); Also Papers I, II, III, IV and V, in the *Proceedings of the Third International Conference on Lie-Admissible Treatments of Irreversible Processes*, C. Corda, Editor, Kathmandu University (2011) published in *Hadronic J.* Vol. 34, Nos.5 & 6 (2011) with the links
http://www.santilli-foundation.org/docs/Isoscattering-I.pdf
http://www.santilli-foundation.org/docs/Isoscattering-II.pdf
http://www.santilli-foundation.org/docs/Isoscattering-III.pdf

6] A.O.E. Animalu *2013 Geno-Bragg's Law and 10*x10 Representation of the SU(3) Symmetry of Quasicrystal Structures, Journal of Computational Methods in Science and Engineering*, 13, 1-17 (2013). See also A.O.E. Animalu 1977 Intermediate Quantum Theory of Crystalline Solids (Prentice-Hall). [7]

A.O.E.Animalu, Nwakanma & Akpojotor 2013 Afr. J. Physics Vol. 6, 1-18,(2013)).

[8] A.O.E. Animalu, Akpojotor, & T.L.Gill in African Journal of Physics Vol.9, p.10-59 (2016)).

[9] A.O.E. Animalu 2019 -Lie-Admissible Approach to "Extended Relativity": Non-Linear Velocity, Mass and Charge Transformations in African *Journal of Phys. Vol.12, p.2-27 (2019) [reformatted from Hadronic J. Vol.10, 321-330(1987];*

[10] P.A.M. Dirac (1970) *Can Equation=of-motion method be used in High Energy Physics?* In Appendix Physics Today April 1970, p. 29.

[11] A.O.E. Animalu and R.M. Santilli SO(2N)xSO(2N) group-theoretic approach to scattering of N-particle (N=5) systems,
http://doi.org/10.52202/059404-0001

[12] O.Y. Ababio, New School Chemistry Certificate Science Series p.123, 131 & 294

13] A.O.E. Animalu "8x8 and 10x10 Hyperspace Representations of SU(3) and 10-fold Point- Symmetry Group of Quasicrystals"
https://www.researchgate.net/publication/258601350

[14] Alexander Animalu, Samuel Edeagu, Godfrey Akpojotor & Erik Trell, Electromagnetism of Atomic Structural Constitution in Deformable Real R3xSO(3) Configuration Space, Afr. J. Phys. 11, 2 (2018).

[15] Brandon Bays www.thejourney.com Einstein's Secret Letter to daughter in 1938 released in1980 on "*Energy = Love x square of speed of light*".

[16] James Cameron's Movie "Titanic" (TikTok@nickygumbel).